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A note on a conjecture from distillability of quantum entanglement

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ABSTRACT

A conjecture from the distillability of quantum entanglement is that when A and B are 4×4 trace zero complex matrices and $\|A\|^2+\|B\|^2 = 1/4$ (where $\|\cdot\|$ is the Frobenius norm), the sum of squares of the largest two singular values of $A\otimes I_4 + I_4 \otimes B$ does not exceed $1/2$. In this paper, the conjecture is proved when

(i) A or B is unitarily similar to a direct sum of 2×2 trace zero matrices;
(ii) A and B are unitarily similar to matrices, when partitioned into 2×2 blocks, having zero diagonal blocks.

1. Introduction

In quantum information theory, a fundamental problem is about the distillability of quantum entanglement [7]. In order to describe the problem, we first introduce some fundamental concepts in quantum information theory. A system is said to be *bipartite* if

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