Hole-flanging of metals and polymers produced by single point incremental forming

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Abstract: This paper presents recent developments in hole-flanging of metal and polymers produced by single point incremental forming (SPIF) to gaining insight into the deformation mechanics of the process and providing guidelines to foster its application in rapid prototyping and small batch production of customised sheet products. Experimentation with stainless steel AISI 304L and polyethylene terephthalate (PET) that comprised mechanical characterisation and utilisation of circle grid analysis in conjunction with the formability limit diagram gives support to the presentation and allows understanding the fundamentals of plastic flow and failure in hole-flanging produced by single point incremental forming and their main differences to hole-flanging produced by conventional pressworking.

Keywords: hole-flanging; single point incremental forming; SPIF; formability; metals; polymers.

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1 Introduction

Hole-flanging produced by pressworking (PW) is a well-known sheet metal forming process in which blanks, with concentric pre-cut holes and the outer peripheries rigidly fixed by blank holders, are forced to shape symmetric (circular) or asymmetric (e.g., rectangular) rims through the employment of punches around dies [Figure 1(a)]. The process is widely utilised in applications where the edges of the holes need to be strengthened, its appearance has to be improved or there is a design requirement to attach tubes or non-round profiles to sheets.

Hole-flanging produced by PW has been extensively studied since the late 1960s and the publications available in the literature allow concluding that the deformation mechanics of the process involve bending and stretching under dominantly uniaxial tensile stresses at the hole edge, and that the maximum allowable strain is influenced by the mechanical properties of the material, the geometry of the hole and flange, the surface quality of the hole edge, the clearance between the punch and die, and the tribology conditions (Yamada and Koide, 1968; Wang and Wenner, 1974; Johnson et al., 1977; Stachowicz, 2008). The process window of hole-flanging produced by PW is characterised by the limiting forming ratio (LFR), above which failure by fracture take place. In case of circular hole-flanges, the LFR is defined as the ratio of the maximum inside diameter $d_{\max}$ of the finished flange to the initial diameter d_0 of the hole.

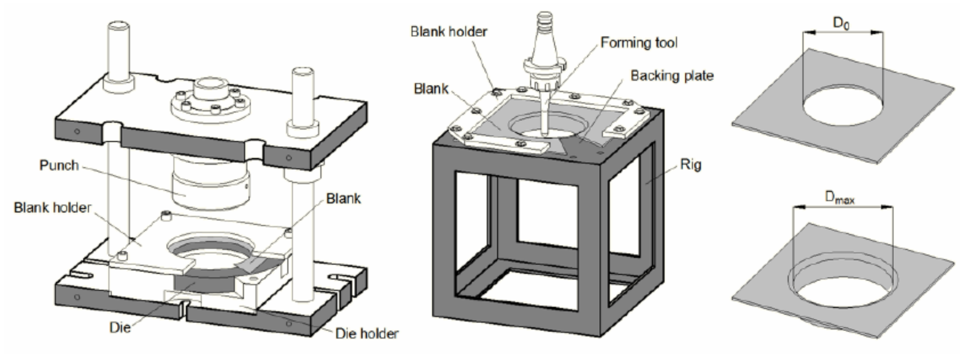
$$LFR = \frac{d_{\max}}{d_0} \tag{1}$$

Hole-flanging produced by PW is utilised in compounded dies in which parts are produced in one press stroke or in progressive dies that carry the parts from one station to the next until the final shape is accomplished. As a result of this, hole-flanging produced by PW requires large batch sizes to return the capital investment in equipment and tooling and is not appropriate for the growing manufacturing trends towards very short life-cycles combined with reduced development and production lead times.

This paper explores the applicability of a newly developed incremental sheet forming process to fabricate hole-flanges in rapid prototyping and small batch production of customised sheet parts. In specific terms, its aims and objectives are focused on recent developments of hole-flanging of metals and polymers produced by multi-stage single point incremental forming (hereafter referred as ‘multi-stage SPIF’) [Figure 1(b)].

Hole-flanging produced by SPIF was developed by Cui and Gao (2010) and makes use of multi-stage tool path deformation strategies (Skjoedt et al., 2010) to produce vertical flange walls (rims) in sheets with pre-cut holes. Over the past four years, the process has been investigated by other researchers in order to understand the influence of backward tool trajectories (Petek et al., 2011), to characterise plastic flow and failure (Centeno et al., 2012; Montanari et al., 2013a) and to determine the critical values of damage and fracture toughness at the onset of fracture (Cristino et al., 2014). The extension of hole-flanging produced by SPIF to commercial polymer blanks at room temperature was recently performed by Silva et al. (2013) and Alkas-Yonan et al. (2014).

Figure 1 Schematic representation of hole-flanging produced, (a) by PW and (b) by SPIF together with a view of (c) the initial blank and final part



(a) (b) (c)

This paper aims at providing a broad view on the deformation mechanics of hole-flanging produced by SPIF. The presentation focus on the differences between plastic flow and failure in hole-flanging produced by PW and SPIF and analyses the formability limits associated with the utilisation of metallic and polymeric blanks. The investigation is supported by an experimental methodology that combines independent characterisation of the mechanical properties of the materials, utilisation of circle grid analysis and forming limit diagrams (hereafter referred as 'FLDs') with the fabrication of hole flanged tests specimens under laboratory controlled conditions.

2 Experimental methods and procedures

2.1 Mechanical characterisation

The work was performed in AISI 304L stainless steel blanks with 0.5 mm thickness and polyethylene terephthalate (PET) blanks with 3 mm thickness. In case of AISI 304L, the specimens utilised in the mechanical characterisation tests were cut out and machined from the supplied sheets at 0º, 45º and 90º degrees with respect to the rolling direction in accordance with the ASTM E8M standard. The mechanical characterisation was performed in an INSTRON 4507 universal testing machine by means of tensile tests and the results are provided in Table 1.

Table 1 Summary of the mechanical properties of the AISI 304L stainless steel sheets

	Modulus of elasticity E (MPa)	*Yield strength σ_Y (MPa)*	*Anisotropy coefficient r*	*Elongation at break A (%)*
0° RD	205,852	273	0.69	40
45° RD	171,513	288	1.06	49
90° RD	190,203	316	0.70	67
Average	184,770	291	0.88	51

In case of PET, the tensile yield stress σ_{YT} and the modulus of elasticity E were determined by means of tensile tests in specimens that were cut out and machined from the supplied sheets in accordance with the ASTM D 638 standard. The compression yield stress σ_{YC} was determined by means of stack compression tests in the abovementioned universal testing machine (Alves et al., 2011) and made use of multi-layer cylinder specimens that were assembled by pilling up circular discs cut from the PET sheets. The results are provided in Table 2.

Table 2 Summary of the mechanical properties of the PET sheets

Modulus of elasticity E (MPa)	*Tensile yield strength σ_{YT} (MPa)*	*Compressive yield strength σ_{YC} (MPa)*
2661.9	48.50	74.39

2.2 Formability limits

The formability limits by necking and fracture [commonly designated as the forming limit curve (FLC) and the fracture forming limit line (FFL)] were determined by means of sheet formability tests covering strain paths from uniaxial to biaxial stretching conditions.

In case of AISI 304L, the procedure utilised for determining the in-plane strains (ε_1, ε_2) involved electrochemical etching a grid of overlapping circles with 2.5 mm initial diameter d on the surface of the initial blanks and measuring the corresponding major a and minor b axes of the ellipses after deformation (Figure 2),

$$\varepsilon_1 = \ln\left(\frac{a}{d}\right) \quad \varepsilon_2 = \ln\left(\frac{b}{d}\right) \tag{2}$$

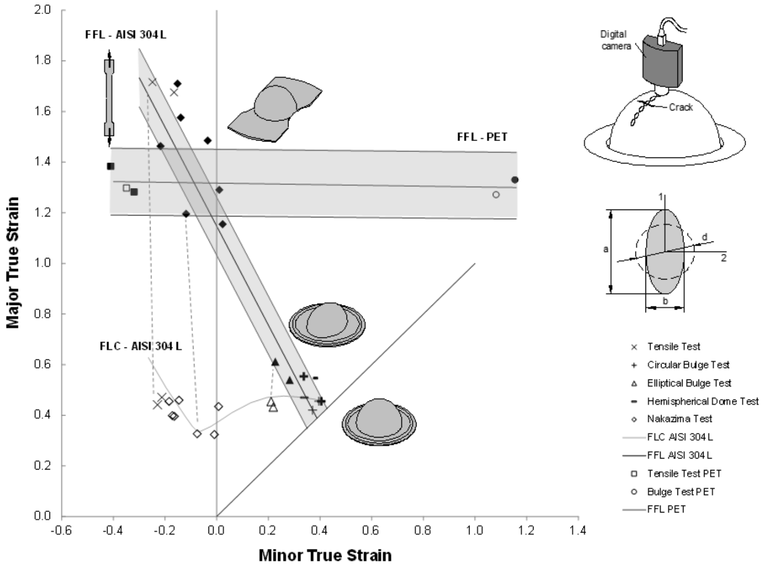
The procedure required the utilisation of a computer-aided measuring system consisting of a 3Com USB camera and the GPA 3.0 software.

The FLC was determined from the experimental values of the in-plane strains (ε_1, ε_2) in the regions placed outside the neck but adjacent to the zone of intense localisation because they represent the condition of uniform thinning at failure by necking. Figure 2 presents the FLC of the AISI 304L sheets.

The FFL of AISI 304L was determined by measuring sheet thicknesses before and after fracture at different locations along the crack in order to obtain the 'gauge length' thickness strains. The gauge length strains along the width direction aligned with the crack were obtained from the initial and deformed reference lengths of two adjacent circles placed close to the crack. The third strain component, locate on the sheet surface with direction perpendicular to the crack, was determined from volume incompressibility. The resulting experimental strains at the onset of fracture are plotted in Figure 2 and were fitted by a straight line (refer to the FFL – AISI 304L in Figure 2) falling from left to right with a slope of '–1.96' and bounded by a grey area that corresponds to an interval of 10% due to the uncertainty in its experimental determination.

In case of PET, it was only necessary to determine the formability limits at fracture (FFL) because neck formation was suppressed. The resulting experimental strains are plotted in Figure 2 and were fitted by a straight line (refer to the FFL – PET in Figure 2) falling from left to right with a slope of '–0.08', which is also bounded by a grey area corresponding to the uncertainty in its experimental determination. The technique utilised for obtaining the FFL of PET was similar to that utilised for AISI 304L and involved printing a grid of circles with 2.5 mm diameter on the surface of the blanks. Further details on the methods and procedures that were utilised for determining the FLC and the FFLs is available in Silva et al. (2011).

Figure 2 Formability limits of the AISI 304l and PET sheets obtained by combining experimental tests and circle grid analysis that are schematically shown in the figure



Note: The solid markers correspond to failure by fracture.

2.3 *Plan of experiments*

The experimental work in hole-flanging of AISI 304L comprised the utilisation of PW and SPIF technologies. The blanks with 250 mm × 250 mm × 0.5 mm were cut out from the supplied sheets, drilled at the centre to deliver holes with different diameters D_0 (refer to Figure 1) and subsequently abraded with medium and fine grit sand papers to eliminate burrs, cracks and to make the edges perpendicular to the surfaces. After being ground, the blanks were electrochemical etched with grids of circles of 2.5 mm in diameter in order to allow measuring the in-plane strains by means of the procedure that was previously described for determining the FLC (refer to Section 2.2).

Table 3 summarises the plan of experiments and includes a schematic representation of the apparatuses that were utilised by PW and SPIF. The PW tests were performed in a 1,200 kN hydraulic press equipped with a tool system consisting of

1 a punch

2 a die

3 a blank holder.

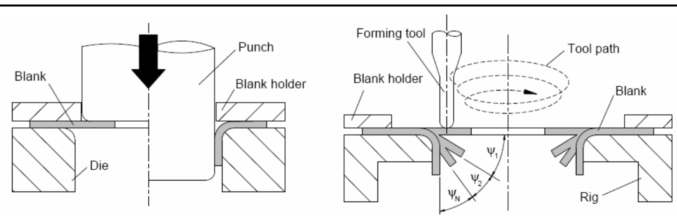
The punch expands the material around the pre-cut hole of the blank into the die cavity to form a hollow flange. The screw-loaded blank holder provides the force to hold-down the blank in position and a draw-bead built at its edge prevents the blank from drawing-in. The punch and die were fabricated from cold working tool steel (120WV4-DIN, hardened and tempered to 60 HRc) and have corner radii of 4 mm.

The SPIF tests were performed on a Deckel Maho DMC63V CNC machining centre and made use of a rig equipped with a backing plate beneath the blank, a forming tool and a screw-loaded blank holder located at the outer perimeter of the blank in order to avoid drawing-in. The forming tool, with a hemispherical tip of 8 mm diameter, was made from cold working 120WV4-DIN tool steel, hardened and tempered to 60 HRc. The blanks were shaped by means of a multi-stage forward tool path strategy that made use of progressively increasing drawing angles from $\psi_1 = 65°$ until $\psi_1 = 90°$ (corresponding to vertical flange walls) with steps of $\Delta\psi = 5°$. The tests were performed with helical tool paths characterised by a step size per revolution equal to 0.2 mm (downward feed) and a feed rate equal to 1,000 mm/min. Lubrication was performed with Castrol Iloform TDN81.

The experimental work in hole-flanging of PET comprised a limited number of SPIF tests that were performed with operating conditions similar to those of AISI 304L (refer to Table 3). Lubrication was performed with a water-soap emulsion in a ratio 1:1.

Table 3 The plan of experiments

(Pre-cut) hole diameter (mm)		*Pressworking (PW)*			*Single point incremental forming (SPIF)*					
		Punch diameter (mm)	*Die opening (mm)*	*Tool diameter (mm)*	*Drawing angle of the intermediate stages ψ_i(°)*					
					1st	*2nd*	*3rd*	*4th*	*5th*	*6th*
AISI 304L	127	146	148.5	8	65	70	75	80	85	90
	121									
	115									
	102									
	95									
	73									
	52									
PET	120	-	-	8						
	102									



3 Results and discussion

Table 4 summarises the results in hole-flanging of AISI 304L produced by PW and SPIF in blanks with different pre-cut hole diameters D_0 (refer to Figure 1). As seen, it is not possible to SPIF hole-flanged parts with vertical walls (ψ = 90º) in AISI 304L blanks with pre-cut hole diameters $D_0 \leq 102$ mm due to failure by fracture.

Table 4 Summary of the results obtained in hole-flanging of AISI 304L produced by conventional PW and SPIF

D_0 (mm)	*Conventional pressworking*	*Single point incremental forming*					
		Drawing angle of the intermediate stages ψ_i(°)					
		65	*70*	*75*	*80*	*85*	*90*
127							
121							
115							
102							
95							
73							
52							

Note: Dark grey cells correspond to failure by fracture.

The above said in conjunction with the fact that the maximum admissible angle was found to decrease from ψ_6 = 90° (corresponding to a hole-flanged part with a vertical wall) when the pre-cut hole diameter D_0 = 115 mm to a much smaller value ψ_2 = 70° when D_0 = 52 mm, allows to conclude that the LFR of SPIF for design purposes is $LFR \cong 1.3$ [refer to equation (1)].

The aforementioned reduction in the maximum admissible drawing angle ψ_i is mainly attributed to the influence of the pre-cut hole diameter D_0. Small holes (such as, for example, D_0 = 52 mm) experience little or no expansion at all and are responsible for approaching the deformation mechanics of hole-flanging to that of truncated conical parts, which fail by circumferential cracking when $\psi_{\max}$ < 75° (Centeno et al., 2012).

The comparison of the LFRs of SPIF and PW allow us to conclude that the latter is more favourable (with nearly 20% increase in the LFR) as it allows producing sound hole-flanged parts from blanks with pre-cut hole diameters $D_0 \geq 95$ mm whereas SPIF is only capable of successfully processing blanks with pre-cut hole diameters $D_0 \geq 115$ mm.

The reason for the advantage of PW over SPIF can be primarily understood by analysing the values of the in-plane strains at fracture that were obtained for two different hole-flange parts produced by PW and SPIF. In fact, as shown in Figure 3, the black circular and square solid markers located in the first and second quadrants of the FLD provide values of the effective strain $\bar{\varepsilon}$ and of the percentage in thickness reduction t_{red} (%), that are significantly more advantageous for the hole-flanged part produced by PW.

$$\bar{\varepsilon} = \frac{1+r}{\sqrt{(1+2r)}}\sqrt{\varepsilon_1^2 + \varepsilon_2^2 + \frac{2r}{(1+r)}\varepsilon_1\varepsilon_2}$$
$$t_{red}(\%) = \left(1 + \exp^{\varepsilon_1+\varepsilon_2}\right) * 100 \qquad (3)$$

Moreover, while the hole-flanged part produced by SPIF with a pre-cut hole diameter D_0 = 102 mm presents a monotonic increase of the in-plane strains (measured along the meridional direction) up to the onset of fracture corresponding to the black solid circular marker located in the first quadrant of the FLD (ψ_4 = 80°), the same part produced by PW presents all the in-plane strains inside the safe region of the FLD, located below the FLC.

In case of SPIF, cracks were found to develop along the circumferential direction of the flange wall because this region of the parts presents values of the in-plane strains that match the critical formability limits of the FFL (refer to D_0 = 102 mm in Figure 3). The hole edges, for example, are subject to much lower values of in-plane strains due to a downward shift of the strain envelope towards biaxial stretching. This downward shift is due to the fact that material placed at the vicinity of the hole edge deforms under combination of plane strain deformation $(d\varepsilon_\phi : d\varepsilon_\theta : d\varepsilon_t = 1:0:-1)$ typical of incremental sheet forming and uniaxial tension $(d\varepsilon_\phi : d\varepsilon_\theta : d\varepsilon_t = -1/2:1:-1/2)$ that results from the boundary conditions associated with the progressive increase of the hole diameter.

On the contrary, hole-flanging produced by PW not only triggers failure in blanks with much smaller diameters (D_0 = 73 mm) as its overall mechanism is due to necking, followed by a change in strain path towards the FFL under plane strain conditions (refer to the vertical line in the second quadrant that connect the solid and the last open square markers). The resulting cracks are aligned in the meridional direction (that is, perpendicular to those found in SPIF) and their morphology is compatible with the existence of uniaxial tension at the hole edge.

Although hole-flanging of AISI 304L produced by SPIF presents a lower performance than PW, it proved successful in producing parts from pre-cut hole diameters $D_0 \geq$ 115 mm (refer, for example, to the results for D_0 = 121 mm in Figure 3). Moreover, the above mentioned relative performance between PW and SPIF can be reversed as a function of the material of the blanks, as shown in other recent publications in the field (Montanari et al., 2013b; Cui et al., 2008).

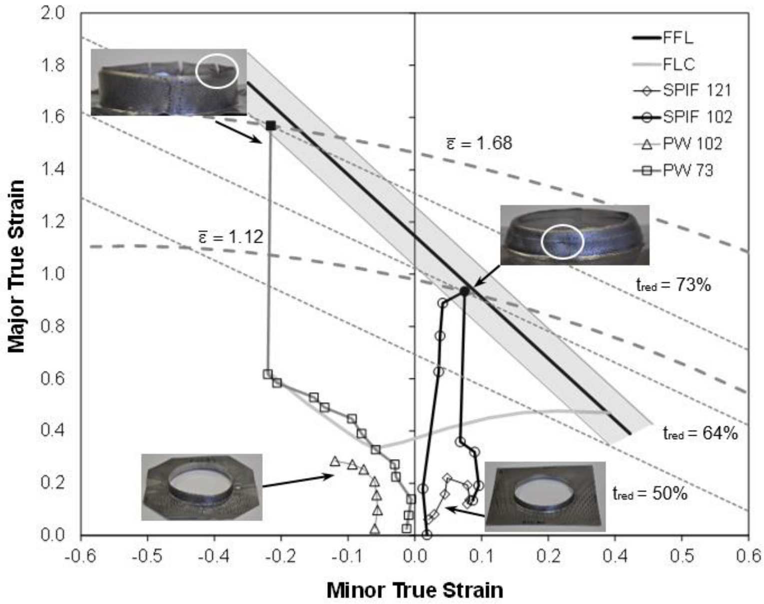
Figure 4 provides the experimental in-plane strains in the FLD for the hole-flanging of PET sheets produced by SPIF at two different stages (first ψ_1 = 65° and final ψ_6 = 90°) of the process. As seen, the strain paths of the first forming stage (ψ_1 = 65°) consist of two different plastic deformation modes:

1 in-plane stretching under plane strain conditions

2 in-plane stretching combined with bending at the hole edge.

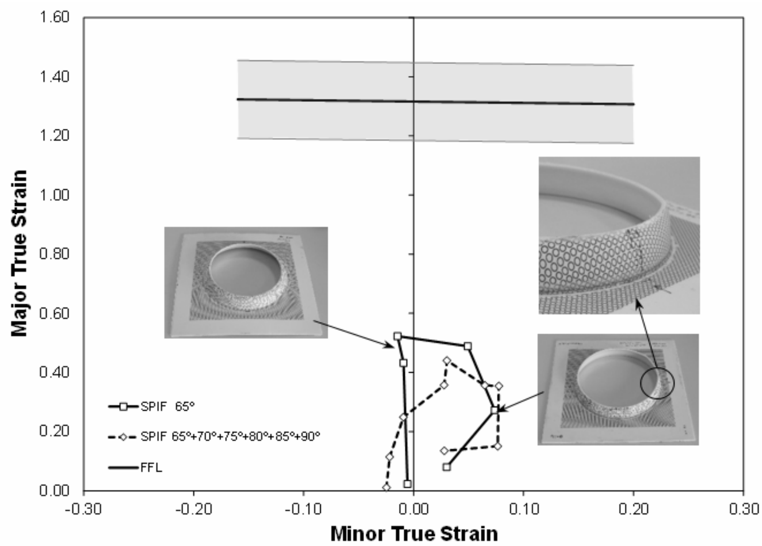
The first plastic deformation mode corresponds to experimental strains that growth along the vertical direction up to ε_1 = 0.52. The second plastic deformation mode corresponds to the reverse strain paths towards bi-axial stretching that are triggered when the forming tool is located at the transition between the inclined wall and the bottom corner radius of the flange adjacent to the hole edge. The reverse strain path has a major influence in plastic flow and failure because it avoids meridional strains to growth towards failure by fracture and, thereby, allows blanks to be successfully shaped into hole-flanged parts with vertical walls.

Figure 3 Representation of the in-plane strains measured along the meridional direction of selected AISI 304L hole-flanged parts fabricated by PW and SPIF in the FLD (see online version for colours)



Note: The solid marker correspond to failure by fracture.

Figure 4 Representation of the in-plane strains measured along the meridional direction of a pet hole-flanged part fabricated by SPIF with a pre-cut hole of 120 mm in the FLD



The reason why PET performs so well at room temperature is attributed to easy plastic deformation due to effective reptation of the polymer chains in its semi-crystalline structure and to high resistance to crack growth due to a value of fracture toughness (R = 31 kJ/m^2) that is close to that commonly found in the aluminium 1,000 series [e.g., R = 53 kJ/m^2 (Cristino et al., 2014)], which are known to be ductile and to provide excellent workability.

4 Conclusions

SPIF is a new sheet forming process with a high-potential for rapid prototyping applications and small-quantity production of metallic and polymer hole-flanged parts. Experimental results and observations showed that formability in hole-flanging produced by SPIF is limited by fracture without previous necking whereas hole-flanging produced by PW is limited by necking followed by fracture. Results also put into evidence the differences in morphology and location of the cracks. SPIF fails by cracking around the circumferential direction at the wall flanges whereas PW fails by meridional cracks at the hole edge. Strain path envelops play a key role in the aforementioned differences between SPIF and PW and the major reason why SPIF does not trigger cracks at the hole edge is because deformation mechanics in this region is responsible for deviation the in-plane strains away from the FFL. The low performance of SPIF compared to PW in case of AISI 304L is attributed to the sharp slope of the FFL that even causes some degree of interaction between necking and fracture in the first quadrant of the FLD. However, the relative performance between hole-flanging produced by PW and SPIF has already been proved in literature to be opposite for other engineering materials.

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