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Parallel Simulation of Cardiac Electrophysiology for a Human Heart

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Introduction

Over the past decades, various models for describing electrical propagation in cardiac tissue have been proposed to understand the underlying phenomena physically and biologically, ranging from single-cell to 3-D full heart models. The models can be used in numerical simulation to reproduce certain physiological and pathological behaviors of the heart, offering quantitative results that have potential applications in pharmacology and medical diagnosis. As a starting point, accurate electrophysiological simulation provides initial conditions for the complete cardiac mechanics modeling of the cardiovascular system. However, the diversity of cellular conductivity leads to tens of equations of response to be solved, which is computationally demanding. Moreover, the real geometry of a human heart introduces extra complexity to the simulation. In this work, we develop parallel solution algorithms based on domain decomposition methods for the cardiac simulation of a human heart using two different ionic models.

Models

The monodomain equations coupled with two ionic models[1][2]

$$\nabla \cdot (\boldsymbol{\sigma} \nabla v) = \chi \left(C_m \frac{\partial v}{\partial t} + I_{ion}(v, \mathbf{w}, \mathbf{z}) \right) - I_s, \qquad \text{in } \Omega \times (0, T],$$

$$\frac{dw_{\zeta_p}}{dt} = \mathbf{G}_\zeta(v, w_{\zeta_p}) = \frac{w_{\zeta_{p\infty}}(v) - w_{\zeta_p}}{\tau_{w_{\zeta_p}}(v)}, \qquad \text{in } \Omega_\zeta \times (0, T],$$

$$\frac{dz_{\zeta_p}}{dt} = \mathbf{F}_\zeta(v, \mathbf{w}_\zeta, \mathbf{z}_\zeta), \qquad \text{in } \Omega_\zeta \times (0, T],$$

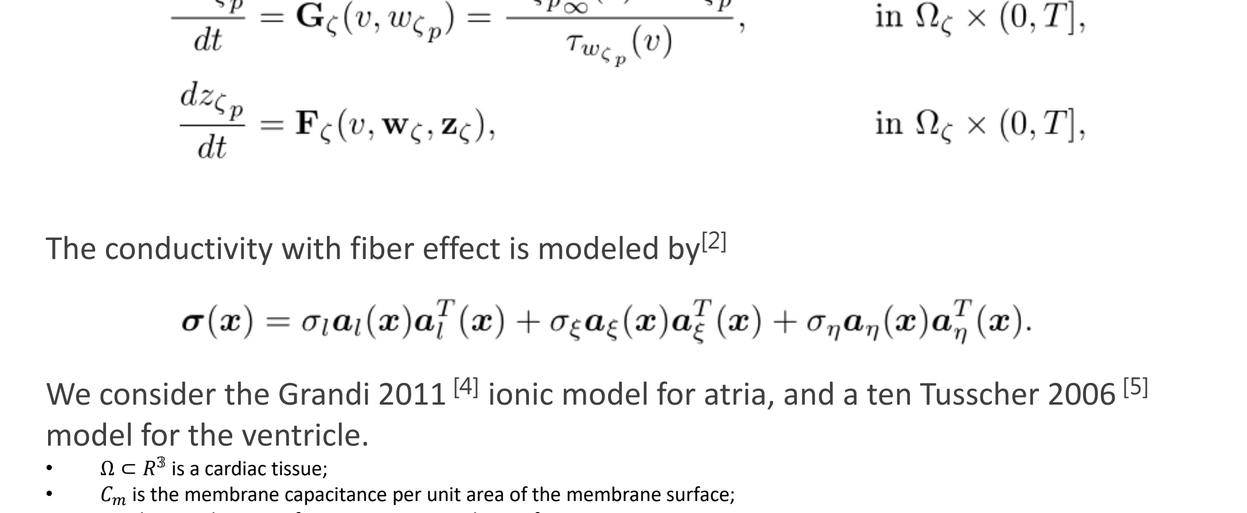
The conductivity with fiber effect is modeled by[2]

$$\boldsymbol{\sigma}(\boldsymbol{x}) = \sigma_l \boldsymbol{a}_l(\boldsymbol{x}) \boldsymbol{a}_l^T(\boldsymbol{x}) + \sigma_\xi \boldsymbol{a}_\xi(\boldsymbol{x}) \boldsymbol{a}_\xi^T(\boldsymbol{x}) + \sigma_\eta \boldsymbol{a}_\eta(\boldsymbol{x}) \boldsymbol{a}_\eta^T(\boldsymbol{x}).$$

We consider the Grandi 2011 [4] ionic model for atria, and a ten Tusscher 2006 [5] model for the ventricle.

- $\Omega \subset R^3$ is a cardiac tissue;
- C_m is the membrane capacitance per unit area of the membrane surface;
- χ is the membrane surface area per unit volume of tissue;
- ζ *can be* $A(atria)$ *or* $V(ventricles)$;
- $w \coloneqq (w_A, w_V), z \coloneqq (z_A, z_V)$;
- w is gating variable, *when* $\zeta =$ A, p∈{m, h, j, d, f , hl, ml, s, xs, xr, x, y, ikurr}, *when* $\zeta =$ V, p∈{ m, h, j, d, f , f2, fcass, r, s, xs, xr1, xr2};
- z is ion concentrations, when $\zeta =$ A, p∈{Nai, Cai, Cajn, Casl, Casr, Najn, Nasl, Csqn, CaM, Myoc, Myom, SLHjn, SLHsl, SLLjn, SLLsl, SRB, TnCHc, TnCHm, TnCl, NaBjn, NaBsl, fCaBjn, fCaBsl, Ki, o, i, ryrr}, when $\zeta =$ V, p∈{Nai, Cai, Casr, Cass, Ki, $\bar{R}$};
- a triplet of orthonormal principal axes $a_l(x), a_\xi(x), a_\eta(x)$, with $a_l(x)$ is parallel to the local fiber direction, $a_\xi(x)$ is tangent to the radial laminae, $a_\eta(x)$ is orthogonal to the radial laminae;
- $\sigma_l, \sigma_\xi, \sigma_\eta$ are the conductivity coefficients measured along the corresponding directions;
- I_{stim} is the externally applied stimulus current.

Methodology



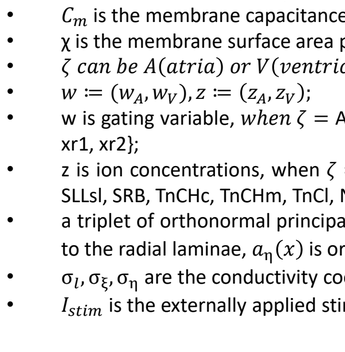


Fig 1: The direction of the cardiomyocytes.

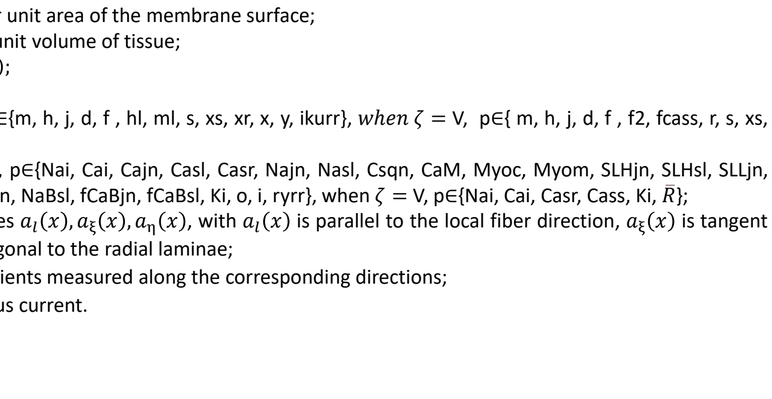
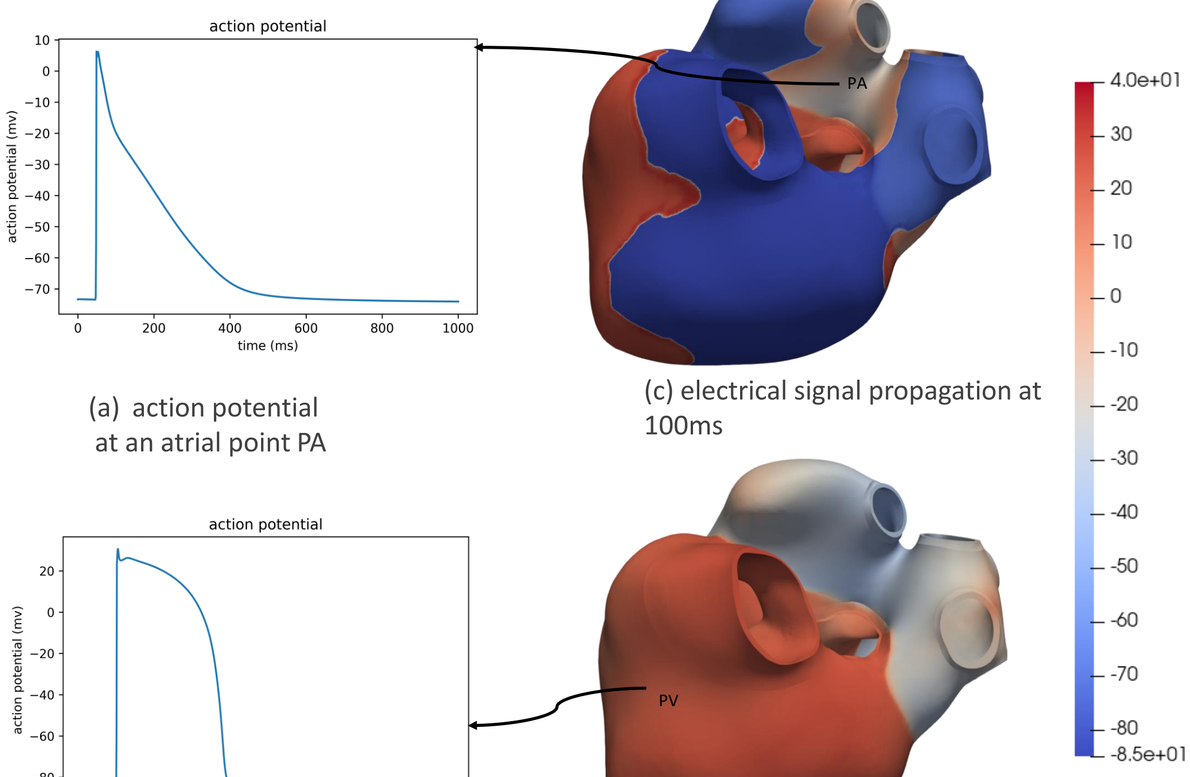


Fig 2: The full heart mesh is partitioned into 16 subdomains using METIS.

Numerical simulation of electrophysiology in a full and healthy heart is carried out using a finite element method on an unstructured mesh. The ionic equations and the monodomain equation are solved in a decoupled way. In addition, we devise a sub-iterative approach that uses the newest gating variables computed by the Rush-Larsen scheme to update the ion concentrations, which improves the efficiency while ensuring the desired accuracy. The linear system is solved by the Conjugate Gradient method with an additive Schwarz preconditioner, offering good scalability.

Numerical Results



(a) action potential at an atrial point PA

(c) electrical signal propagation at 100ms

(b) action potential at a ventricular point PV

(d) electrical signal propagation at 200ms

Fig 3: Simulation results of action potential and electrical signal propagation.

np	Time(s)	Speed up	Parallel Efficiency
2	2.96×10^6	/	/
4	1.54×10^6	1.92	96.2%
8	8.04×10^5	3.68	92.1%
16	4.57×10^5	6.48	81.0%

Tab 1: : Strong scalability test during a heartbeat cycle with Δx = 0.5mm and Δt = 0.1ms, np is the number of processor cores.

Findings/Conclusions

- Cardiac electrophysiology behaviors are reproduced by numerical simulation of a full patient-specific heart with anisotropic tissue and cardiomyocytes.
- A parallel domain decomposition method is used for solving the resulting large linear systems.
- Results of a strong scalability test indicate that the parallel algorithm has good parallel efficiency.

References

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