Quaternion Singular Spectrum Analysis of Pupillary Dynamics for Health Monitoring

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2022 IEEE 4th Eurasia Conference on IOT, Communication and Engineering (ECICE) | 978-1-6654-8208-0/22/$31.00 ©2022 IEEE | DOI: 10.1109/ECICE55674.2022.10042889

***Abstract*—Pupillary dynamics have been monitored in mobile health for clinical diagnosis and health evaluation. However, such signals captured in a mobile scenario would be susceptible to noise from the surrounding environment. Pupil size data also consist of missing data due to blinks. In addition to pupil size, researchers have suggested including multimodal information such as gaze and head movements for enhanced monitoring cognitive and neural activities. Multichannel signal processing is required for the simultaneous processing of these signals, while their mutual relations are exploited. This paper explores a novel multichannel signal processing technique, quaternion singular spectrum analysis (QSSA), for processing eye-movement data, which include pupil diameter and gaze positions. The performance based on real data is compared with traditional methods. This is a novel application of QSSA in processing eye movement data for mobile health.**

Keywords—Quaternion Singular Spectrum Analysis, Multichannel Signal Processing, Pupillary Dynamics, Health Monitoring, Mobile Health

I. Introduction

As the eyes can reflect the health condition and the underlying bodily functions, measurement and analysis of ocular structures, eye movements, and pupillary dynamics have been conducted in mobile health for clinical diagnosis and health evaluation [1]. Pupillary dynamics analysis with a smart eyewear can become a new mobile health monitoring application [2]. However, there are technical challenges that need to be tackled before it can be practically realized. First, eye dynamics captured in a mobile scenario would be susceptible to noise from surrounding environment and internal factors such as age and emotion. In an unconstrained setting, the most influential determinant of pupil size is the brightness level of the environment. Traditional noise filtering methods such as moving average filtering [3], median filtering [4] and Savitzky-Golay filtering [5] are commonly employed to remove noise and artifacts of pupillary signals. However, these filters suffer from signal distortion. Sudden deviations are over smoothed, and consequently these filters may lead to physiological parameters being underestimated [6]. On the other hand, singular spectrum analysis (SSA), a subspace separation method, has been employed in biosignal denoising [7]. It has superior performances compared to conventional linear time-frequency analysis techniques [8]. SSA can effectively denoise a signal, while maintaining the signal trend without introducing signal decay or distortion [9].

Secondly, eye movement always consists of missing data caused by blinking. Researchers have used methods such as elimination of events or interpolation to tackle the problem [10]. However, the elimination method leads to data loss and attenuation of statistical power, making it difficult for subsequent analysis. As eye blinks are characterized by a sudden decrease followed by sudden increase in pupil size, changes of pupil size cannot be fully removed using interpolation, thus creating noise artifacts in the pupil size signal [11]. On the other hand, SSA algorithm can be used to have spectral characteristics preserved estimation of missing data. Based on iterative SSA, an algorithm has been developed for filling of missing data during blinking events [12]. It was verified that the trend of the signal could be faithfully reconstructed.

In mobile health with smart eyewear, eye movement data are collected in an unconstrained setting while participants engage in instrumental activities of daily living (IADL) like driving and doing household chores [13]. Clinical studies have shown that impairment in the performance of IADL corresponds to early stage of mild cognitive impairment [14]. In contrast to having only pupil size being monitored for diagnosis, researchers have suggested including multimodal data for better tracing of cognitive and neural deficits [15]. Apart from pupillary dynamics, other information, such as gaze positions, head movements, and other biosignals should be employed for feature extraction and further analysis to holistically reflect mental health status. A multichannel signal processing technique is required for the simultaneous processing of multiple channels, and for exploiting the mutual relationships between the channels. The traditional method mainly manipulates each data channel separately and it cannot extract and exploit the mutual relation among different channels. Recently, quaternion SSA (QSSA) was shown to be a more promising way to account for the multichannel correlation. Since the work of Enshaeifar et al. [16], QSSA has been applied in many fields, for example, color image denoising [17], fault diagnosis [18], sleep-stage classification [19], and seismic data processing [20]. QSSA was found to have better performance as compared with traditional separate channel processing techniques.

This paper aims to explore a novel multichannel signal processing technique by using QSSA for processing multichannel eye-movement data, including pupillary dynamics and gaze positions, in order to tackle the above-mentioned problems. Sections II and III cover background of the SSA algorithm and quaternion valued algebra. Augmented QSSA is then introduced in Section IV. The methodologies and results of the application in missing data filling and signal denoising are shown in Sections V and VI. Section VII presents the results and future work, and Section VIII gives a conclusion.

II. SINGULAR SPECTRUM ANALYSIS

SSA is a non-parametric and data driven technique for time-series analysis. Its ability to provide meaningful results without assumption of the data makes it a useful tool in real-world applications. SSA works by decomposes a signal into independent and interpretable components. Some of the components are selected and grouped for reconstructing a denoised signal. SSA consists of two main stages [7, 19]. The first stage decomposes the sequence into components. Let $x = [x_1 \cdots x_N]^T \in \Re^N$ be the vector of a one dimensional sequence with length N, and let L be the window length, and $1 < L < \frac{N}{2}$. Let $K = N - L + 1$. Let $S \in \Re^{L\times K}$ be the trajectory matrix as follows:

$$S = \begin{bmatrix} x_0 & x_1 & x_2 & \cdots & x_{K-1} \\ x_1 & x_2 & x_3 & & x_K \\ \vdots & & & \ddots & \vdots \\ x_{L-1} & x_L & x_{L+1} & \cdots & x_{N-1} \end{bmatrix} \in \Re^{L\times K}$$

Let the triplets, U, V and $\wedge^{\frac{1}{2}}$ respectively be the left and right eigenvectors, and the singular values resulting from the singular value decomposition (SVD) on S. That is,

$$S = U \wedge V^T = \sum_{r=1}^{R} \sigma_r u_r v_r^T = \sum_{r=1}^{R} S_r$$

, where $R = rank(S) \le L$. Then, we have $SS^T = U \wedge U^T$.

Constructing the denoised signal in the second stage involves first categorizing the components into meaningful groups, then performing diagonal averaging. S_r is categorized into the group of different subspaces components and the group of noise components.

Reconstruct the denoised signal involves retaining the underlying subspaces' components, and discarding the noise components, which have smaller singular values. Let W_s be the index set corresponding to the underlying subspaces' components. Let $\hat{S}_s$ be the sum of all S_j for all j in W_s. That is,

$$\hat{S_s} = \sum_{j\in W_s} S_j = \begin{bmatrix} \hat{s}_{00} & \hat{s}_{01} & \cdots & \hat{s}_{0,K-1} \\ \hat{s}_{10} & \hat{s}_{11} & & \hat{s}_{1,K} \\ \vdots & & \ddots & \vdots \\ \hat{s}_{L-1,0} & \hat{s}_{L-1,L} & \cdots & \hat{s}_{L-1,N-1} \end{bmatrix}$$

Then $\hat{S}_s$ is converted to a one dimensional vector through diagonal averaging. Let $\hat{x} = [\hat{x}_1 \cdots \hat{x}_N]^T$ be the one dimensional reconstructed vector. Then, we have:

$$\begin{cases} \hat{x}_1 = \hat{s}_{00} \\ \hat{x}_2 = (\hat{s}_{01} + \hat{s}_{10})/2 \\ \hat{x}_3 = (\hat{s}_{02} + \hat{s}_{11} + \hat{s}_{20})/3 \\ \cdots \end{cases}$$

III. QUATERNION VALUED ALGEBRA

Quaternions are 4-D hypercomplex numbers. A quaternion $x \in \mathbb{H}$ is defined as $x = r_1 + ir_i + jr_j + kr_k$, where r_1, r_i, r_j, r_k are all real numbers, and the imaginary units (i, j, k) have the following properties:

$$ij = k = -ji$$
$$jk = i = -kj$$
$$ki = j = -ik$$
$$i^2 = j^2 = k^2 = ijk = -1.$$

Quaternions therefore form a noncommutative normed division algebra $\mathbb{H}$, that is, in general for $x, y \in \mathbb{H}, xy \neq yx$. The conjugate of a quaternion x is $x^* = r_1 - ir_i - jr_j - kr_k$, and η is a pure unit quaternion iff $\eta^2 = -1$. The involution of a quaternion x over a pure unit quaternion η is $x^{(\eta)} = -\eta x^* \eta$. It represents the reflection of x over the plane spanned by $\{1, \eta\}$. Three involutions $x^{(i)}, x^{(j)}, x^{(k)}$ of x corresponding to i, j, k components are defined as:

$$x^{(i)} = r_1 - ir_i + jr_j + kr_k$$

$$x^{(j)} = r_1 + ir_i - jr_j + kr_k$$

$$x^{(k)} = r_1 + ir_i + jr_j - kr_k$$

To exploit correlation between data channels, augmented statistics can be used to incorporate the complementary covariance matrices and to exploit second-order information [16]. The augmented quaternion vector, $x_a = \left[x^T, x^{(i)^T}, x^{(j)^T}, x^{(k)^T}\right]^T$ is used to find the augmented covariance matrix as follows:

$$R_{x_a,x_a} = \begin{bmatrix} R_{x,x} & R_{x,x^{(i)}} & R_{x,x^{(j)}} & R_{x,x^{(k)}} \\ R^{(i)}_{x,x^{(i)}} & R^{(i)}_{x,x} & R^{(i)}_{x,x^{(k)}} & R^{(i)}_{x,x^{(j)}} \\ R^{(j)}_{x,x^{(j)}} & R^{(j)}_{x,x^{(k)}} & R^{(j)}_{x,x} & R^{(j)}_{x,x^{(i)}} \\ R^{(k)}_{x,x^{(k)}} & R^{(k)}_{x,x^{(j)}} & R^{(k)}_{x,x^{(i)}} & R^{(k)}_{x,x} \end{bmatrix},$$

, where the covariance matrix $R_{x,x} = Exx^H$ and the three complementary covariance matrices $R_{x,x^{(i)}} = Exx^{(i)^H}$, $R_{x,x^{(j)}} = Exx^{(j)^H}$ and $R_{x,x^{(k)}} = Exx^{(k)^H}$ can be identified. Because R_{x_a,x_a} involves both the covariance and pseudo-covariance matrices, the complete second order information in the quaternion-valued data are incorporated.

IV. QUATERNION SINGULAR SPECTRUM ANALYSIS

SSA can be performed on a quaternion valued signal as follows [19]. Let S_a, S_b, S_c and S_d be the real valued components, and the $i, j, and\ k$ imaginary valued components of the trajectory matrix S, respectively. That is, $S = S_a + iS_b + jS_c + kS_d$. Likewise, $S^{(i)}, S^{(j)}$ and $S^{(k)}$ are the i, j, and k involutions of S, respectively. That is,

$$S^{(i)} = -i\bar{S}i = S_a - iS_b + jS_c + kS_d$$

$$S^{(j)} = -j\bar{S}j = S_a + iS_b - jS_c + kS_d$$

$$S^{(k)} = -k\bar{S}k = S_a + iS_b + jS_c - kS_d$$

Let $S^a = \left[S^T\ S^{(i)^T}\ S^{(j)^T}\ S^{(k)^T}\right]^T \in \mathbb{H}^{4L\times K}$, then, the quaternion covariance matrix R_{S_a,S_a} can be denoted as:

$$S^a S^{aH} = \begin{bmatrix} R_{S,S} & R_{S,S^{(i)}} & R_{S,S^{(j)}} & R_{S,S^{(k)}} \\ R^{(i)}_{S,S^{(i)}} & R^{(i)}_{S,S} & R^{(i)}_{S,S^{(k)}} & R^{(i)}_{S,S^{(j)}} \\ R^{(j)}_{S,S^{(j)}} & R^{(j)}_{S,S^{(k)}} & R^{(j)}_{S,S} & R^{(j)}_{S,S^{(i)}} \\ R^{(k)}_{S,S^{(k)}} & R^{(k)}_{S,S^{(j)}} & R^{(k)}_{S,S^{(i)}} & R^{(k)}_{S,S} \end{bmatrix} \in \mathbb{H}^{4L\times 4L},$$

Through quaternion valued singular value decomposition (QSVD) on S^a, we get:

$$S^a = U \wedge^{1/2} V^H = \sum_{r=1}^{4L} \sqrt{\sigma_r} u_r v_r^H = \sum_{r=1}^{4L} S_r^a$$

$$S^a S^{aH} = U \wedge U^H$$

S_r^a are categorized into finite number of groups. Then in each group, diagonal averaging of the sum of the components is performed. Let W_s be the index set corresponding to the underlying subspaces' components. Let $\hat{S}_s{}^a$ be the sum of all S_j^a for all j in W_s. That is,

$$\hat{S}_s{}^a = \sum_{j\in W_s} S_j^a = \begin{bmatrix} \hat{s}_{11} & \hat{s}_{12} & \cdots & \hat{s}_{1,K} \\ \hat{s}_{21} & \hat{s}_{22} & & \hat{s}_{1,K+1} \\ & \vdots & \ddots & \vdots \\ \hat{s}_{L,1} & \hat{s}_{L,L+1} & \cdots & \hat{s}_{L,N} \end{bmatrix}$$

Let $\hat{x} = [\hat{x}_1 \cdots \hat{x}_N]^T$ be the one dimensional reconstructed vector. Through diagonal averaging on $\hat{S}_s{}^a$, we have:

$$\begin{cases} \hat{x}_1 = \hat{s}_{11} \\ \hat{x}_2 = (\hat{s}_{12} + \hat{s}_{21})/2 \\ \hat{x}_3 = (\hat{s}_{13} + \hat{s}_{22} + \hat{s}_{31})/3 \\ \cdots \end{cases}$$

V. METHODOLOGIES

In this section, the QSSA multichannel signal processing technique is applied on pupillary dynamic processing. It involves blinks gap filling and signals denoising in an unconstrained setting. The pupil size dataset was provided by Kret et al. [22]. The data was acquired with Eyelink system from SR Research Limited.

A. Blinks Gap Filling

To test the blinks gap filling performance, 6 samples of pupil-size data were chosen as shown in Figure 1. A total of 14 equal interval length (1000 time samples) segments without blinks were selected. 17 blinks per segment starting from 100th time sample to 900th time-sample were generated. For each generated blink, the length of data missing was 50 time samples, during which the pupil-size values were set to zero. In total, there are 238 blinks for performance evaluation. After generating the blink samples, iterative SSA was performed to fill in the gaps by using the reconstructed signal based on the first three decomposed components.

In each iteration, the mean square error (MSE) of the filled segment as compared with the previous iteration was computed. If the MSE was lower than a stopping criteria, such as 1×10^{-2} , or if the number of iteration was more than a preset limit, for example, 60, the SSA algorithm would stop and the filled value of the last iteration would be the result of blink gap filling. Figure 2 illustrates an example of blink gap filling by iterative SSA. The red lines show the filled results from different iterations. To compare the performance of the iterative SSA algorithm, a 4-points cubic spline interpolation was employed. Meanwhile, white Gaussian noise having different signal to noise ratio (SNR) was added to the pupil-size data for better simulation of the real environment in mobile scenario. For each SNR, 50 trials were conducted, and the mean MSE was calculated for comparison.

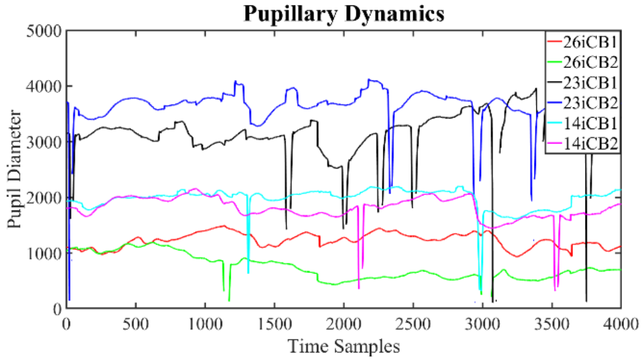

Fig. 1. The pupil size data of each sample.

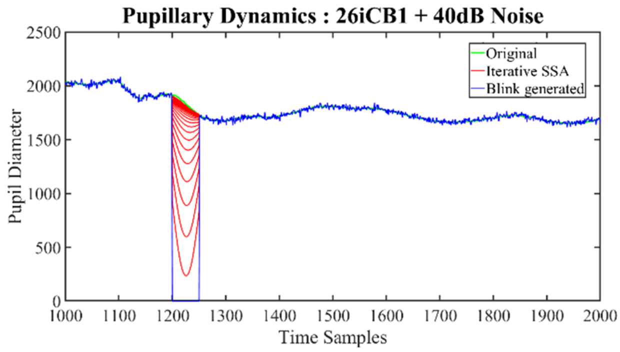

Fig. 2. Example of iterative SSA on segment of sample "26iCB1" with 40dB additive Gaussian noise. The green line is the original pupil-size data. The blue line shows the blink being generated by setting a length of 50 time-samples to zero value. The red lines show the filled values of different iterations.

To handle multichannel eye-movement data including pupil diameter, gaze x and y positions, iterative QSSA was implemented. Per iteration, these 3 signals were combined to form a pure quaternion (only imaginary parts) and only the first component was used for reconstruction. After converting the reconstructed quaternion signal into separate real-valued signals, the blink gap was filled with the reconstructed data in the corresponding signals.

B. Denoising

As mentioned in Section I, for multichannel signal processing, QSSA was also be proven to have better performance for denoising but without quantitative analysis. Here, the denoising performance of QSSA is evaluated as compared with common single-channel filtering techniques such as median filtering and Savitzky-Golay filtering. Meanwhile, the importance of the number of components being selected based on singular values and their relation to denoising performance will also be studied. In the experiment, 4 pupil-size data samples are selected from Kret's dataset.

The missing data in the samples were filled by iterative QSSA as mentioned previously. A white Gaussian noise was added to the gap-filled data with different SNR level from 10dB to 40dB for performance evaluation. Then, the noisy signals were handled by the above methods separately. MSE between the denoised signals and original signals was computed in order to evaluate denoising performance. In QSSA, the four signals were first combined into a quaternion valued signal, and went through the procedures as mentioned in Section IV. The quaternion valued signal was decomposed into signal part and noisy part according to the singular values, only signal part was reconstructed to form the denoised signals.

VI. RESULTS

A. Blinks Gap Filling

Figure 3 shows an example of blink gap filling. The iterative SSA's result (red line) was closer to the original signal (green line) as compared with the one by cubic-spline interpolation (cyan line). The reconstruction signal from SSA included the trend and periodic components of the original signal which preserved the overall structure of the data much better than cubic spline interpolation. Table I presents the overall results. When the SNR is high, as SNR=50dB, there are 169 samples (71% of total no. of samples) in which MSE between iterative SSA reconstructed signal and the original signal, denoted as MSE_{ISSA}, is less than the MSE between cubic spline interpolated signal and the original signal, denoted as MSE_{CS}. As seen in the table, the iterative SSA method outperforms cubic-spline interpolation, especially when the SNR is low. Figure 4 shows an example of blink gap filling in multichannel eye-movement data by iterative QSSA. As seen in the figure, there are abrupt changes before and after the blink, QSSA not only fills the gap with preserving overall structure but also removes artifacts caused by blinking.

TABLE I. BLINK GAP FILLING PERFORMANCE EVALUATION

Total 238 samples for evaluation	**No. of samples (%)**	
	MSE_{ISSA} [a] < MSE_{CS} [b]	**MSE_{CS} < MSE_{ISSA}**
SNR = 50dB	169 (71%)	69 (29%)
SNR = 40dB	222 (93.3%)	16 (6.7%)
SNR = 30dB	237 (99.6%)	1 (0.4%)

a. MSE_{ISSA} denotes the MSE between the iterative SSA reconstructed signal and the original signal.
b. MSE_{CS} denotes the MSE between the cubic spline interpolated signal and the original signal.

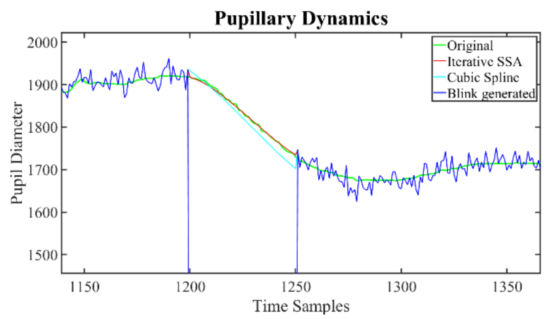

Fig. 3. An example of the blink gap filling test. The iterative SSA result denoted as red line is more close to the original pupil-size data (green line) as compared with the cubic spline interpolation (cyan line). The MSE of iterative SSA and cubic spline interpolation are 10.5648 and 276.3463 in this trial.

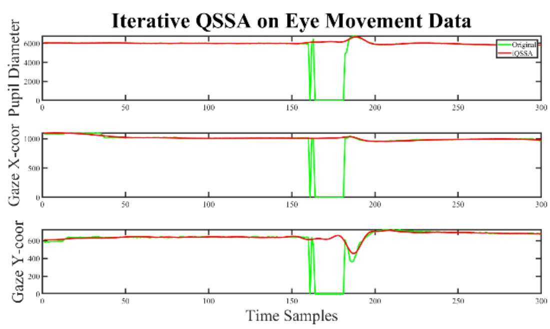

Fig. 4. Iterative QSSA applied on multichannel eye-movement data, including pupil diameter, gaze x and y positions.

B. Denoising

As shown in Figure 5, the denoising performance of QSSA outperforms other methods especially for small SNR. Furthermore, when the SNR=10dB, as shown in Figure 6, the MSE of QSSA is found to have a minimum around the turning point of singular values. That means the proper number of components for separation of the signal part and the noise part only based on the singular values can be easily identified. However, for large SNR, when SNR=40dB as shown in Figure 7, the selection of number of components are critical for the denoising performance but the minimum MSE of QSSA cannot be easily identified by singular values only as in small SNR. Therefore, additional criteria are needed for proper selection.

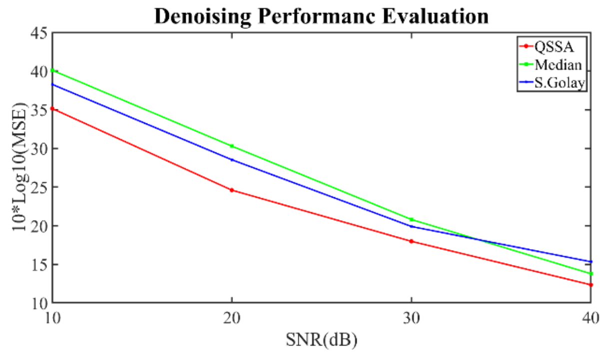

Fig. 5. The denoising performance of QSSA over different SNRs is evaluated to outperform other methods, especially for small SNR.

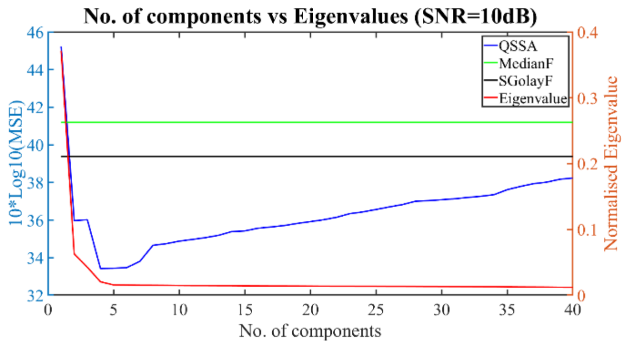

Fig. 6. For small SNR, e.g. in this case SNR=10dB, the MSE of QSSA is found to have a minimum around the turning point of eigenvalue.

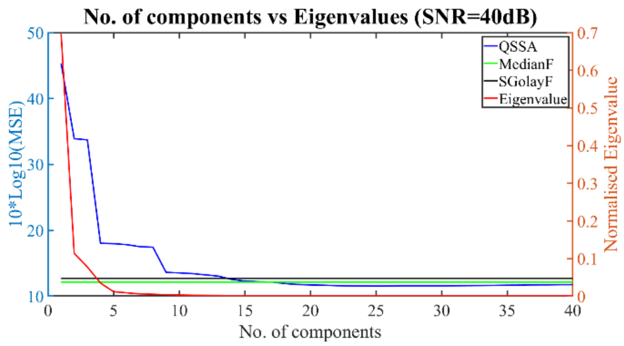

Fig. 7. For large SNR, in this case SNR=40dB, the relation between the number of components and eigenvalues is not easily interpreted.

VII. DISCUSSION

Although the performance of applying QSSA in blinks gap filling and denoising for multichannel eye-movement signal processing is better than traditional separate-channel methods, QSSA also has shortcomings in real implementation. First, the singular values cannot be a universal method to select the number of components for subspace separation. One promising method is to analyze the spectral properties of singular vectors for better subspaces separation [7]. As SSA can be interpreted as a filtering process and the singular vectors can be viewed as a bank of eigenfilters, similarly for QSSA as quaternion valued eigenfilters, the spectral properties of these eigenfilters reflects valuable information for subspaces separation as well as feature extraction. More studies should be conducted [23-24].

One of the main drawbacks of QSSA is the long computation time [18]. The heavy computation loading leads to high demand for computer performance. The heaviest burden in QSSA is the QSVD operation which involves complex quaternion valued matrix decomposition. Recently, a Lanczos-based method has been proposed to enhance computation efficiency of partial SVD triplets of large-scale quaternion matrices [25]. This method is shown to have better performance as compared with state-of-the-art methods theoretically. Its practical implementation was shown to reduce the computation time dramatically as compared with single channel operation [18]. Therefore, by using Lanczos-based QSVD, QSSA will be more computational efficient and suitable for real-time processing.

VIII. CONCLUSION

This paper provides a preliminary exploration of QSSA, a novel multichannel signal processing technique, in analyzing pupillary dynamics for mobile health. Real pupil size data are used to investigate the performance in the application of blinks gap filling and signals denoising. The results show that QSSA outperforms traditional methods. However, these results are preliminary and a validation on a larger dataset is needed. Suggestions for the enhancement of applying QSSA have been proposed. One way is to have better criteria for subspaces separation and feature extraction by exploring the spectral properties of quaternion valued eigenfilters. Another way is to employ Lanczos-based QSVD method to shorten the computation time. To the best of our knowledge, this is the first application of QSSA on pupillary dynamics data for mobile health monitoring.

ACKNOWLEDGMENT

The work described in this paper was fully supported by a grant from the Research Grants Council of the Hong Kong Special Administrative Region, China (UGC/FDS16/E15/21).

REFERENCES

[1] D.L. Chandler, "Eye as a window to health: albeit slow, research is progressing on contact lenses for medical diagnosis," IEEE Pulse, vol.9, no.6, pp. 20-23, 2018.

[2] K. Hung, "Pupillary Dynamic Monitoring for Mobile Health," in Proc. 39th Annual Conference of the IEEE IECON, Vienna, Austria, 2013.

[3] R.A. Warman, S. Wibirama and A. Bejo, "Performance comparison of signal processing filters on smooth pursuit eye movements," ICITISEE pp.111-115, 2017.

[4] H.C. Huang and T.C.M. Lee, "Data adaptive median filters for signal and image denoising using a generalized SURE criterion," IEEE Signal processing letters, vol.13, no.9, pp.561-564, 2006.

[5] M. Nystrom and K. Holmqvist, "An adaptive algorithm for fixation, saccade, and glissade detection in eyetracking data," Behavior Research Methods, vol.42, no.1, pp.188-204, 2010.

[6] W.W, Dai, I. Selesnick, J.R. Rizzo, J. Rucker and T. Hudson, "A nonlinear generalization of the Savitzky-Golay filter and the quantitative analysis of saccades," J. of Vision, vol.17, no.9, pp.1-15, 2017.

[7] N. Golyandina, V. Nekrutkin, A.A. Zhigljavsky, Analysis of time series structure: SSA and related techniques, Google eBook, 2010.

[8] M. Azarbad, H. Azami, S. Sanei and A. Ebrahimzadeh, "A time-frequency approach for EEG signal segmentation," J. of AI and Data Mining, vol.2, no.1, pp.63-71, 2014.

[9] Y. Lu, X. Zhang, L.W. Jing, X.Q. Li and X.G. Fu, "Estimation of the foetal heart rate baseline based on singular spectrum analysis and empirical mode decomposition," Future Generation Computer Systems, vol.112, pp.126-135, 2020.

[10] S. Mathot, J. Fabius, E.V. Heusden and S.V. Stigchel, "Safe and sensible preprocessing and baseline correction of pupil-size data," Behavior Research Methods, vol.50, pp.94-106, 2018.

[11] R. Hershman, A. Henik and N. Cohen, "A novel blink detection method based on pupillometry noise," Behavior Research Methods, vol.50, pp.107-114, 2018.

[12] F. Onorati, M. Mauri, V. Russo and L.T. Mainardi, "Reconstruction of pupil dilation signal during eye blinking events," Proceedings VII International Workshop on Biosignal Interpretation, 2012.

[13] M. Sheikh, M. Qassem and P.A. Kyriacou, "Wearable, environmental, and smartphone-based passive sensing for mental health monitoring," Frontiers in Digital Health, vol.3, article 662811, 2021.

[14] S.C. Seligman and T. Giovannetti, "The potential utility of eye movements in the detection and characterization of everyday functional difficulties in mild cognitive impairment," Neuropsychol Rev., vol.25, pp.199-215, 2015.

[15] R.Z. Marandi and P. Gazeranti, "Aging and eye tracking: in the quest for objective biomarkers," Future Neurology, vol.14, no.4, 2019.

[16] S. Enshaeifar, S. Kouchaki, C.C. Took and S. Sanei, "Quaternion singular spectrum analysis of electroencephalogram with application in sleep analysis," IEEE Trans. Neural Syst. Rehabil. Eng., vol.24, no.1, pp.57-67, 2016.

[17] Y. Lin, B.W.K. Ling, N. Xu and X.L. Zhou, "Two dimensional quaternion valued singular spectrum analysis with application to image denoising," J. of the Franklin Institute, vol.359, pp.3808-3830, 2022.

[18] Y.L. Ma, J.S. Cheng, P. Wang, J. Wang and Y. Yang, "A novel Lanczos quaternion singular spectrum analysis method and its application to bevel gear fault diagnosis with multi-channel signals," Mechanical Systems and Signal Processing, vol.168, 2022.

[19] Z. Huang and B.W.K. Ling, "Sleeping stage classification based on joint quaternion valued singular spectrum analysis and ensemble empirical mode decomposition," Biomedical Signal Processing and Control, vol.71, 2022.

[20] B. Bahia and M.D. Sacchi, "Quaternionic rank-reduction methods for vector-field seismic data processing," Digital Signal Processing, vol.87, pp.178-189, 2019.

[21] T.A. Ell and S.J. Sangwine, "Quaternion involutions and anti-involutions," Computers and Mathematics with Applications, vol.53, no.1, pp.137-143, 2007.

[22] M.E. Kret and E.E.S. Sjak-Shie, "Preprocessing pupil size data: guidelines and code," Behavior Research Methods, vol.51, pp.1336-1342, 2019.

[23] T.J. Harris and H. Yuan, "Filtering and frequency interpretations of Singular Spectrum Analysis," Physica D vol.239, pp.1958-1967, 2010.

[24] A.M. Tomé, A.R. Teixeira, N. Figueiredo, I.M. Santos, P. Georgieva and E.W. Lang, "SSA of biomedical signals: a linear invariant systems approach," Statistics and its Interface, vol.3, pp.345-355, 2010.

[25] Z. Jia, M.K. Ng and G.J. Song, "Lanczos method for large-scale quaternion singular value decomposition," Numerical Algorithms, vol.82, pp.699-717, 2019.