Prediction of Ground Surface Settlements by Analytical Method and Finite Element Model

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ABSTRACT: The soil surrounding the excavation of shallow tunnels may experience destructive deformations in the vicinity of the tunnel. Particularly in densely populated areas, where land resources are scarce, the ground subsidence caused by tunnel excavation, which in turn affects the surrounding structures, is a phenomenon that deserves more attention. Over the past few decades, various research methods for predicting the maximum vertical ground surface settlements have been proposed by dissimilar researchers in their scientific literature. In the paper, an empirical estimation method proposed by Peck (1969), an analytical technique for evaluating ground movements proposed by Pinto (2014), and a numerical simulation method accomplished by Plaxis 2D, a finite element model software, are selected as prediction approaches of ground surface settlements. The monitoring subsidence measurements are from a project of a metro tunnel excavated by an earth pressure balance shield tunnel boring machine constructed in a dense sandy gravel layer. After the comparison of monitoring deformations with the estimation results calculated by different prediction approaches, the maximum vertical settlement of surface ground evaluated by the analytical technique and finite element model has great agreements with the actual measured values, which has been proved that these two prediction methods will contribute to the conservation of nearby structures during the construction of underground infrastructures.

KEYWORDS: Tunnels, Settlements, Analytical techniques, and FEM.

1. INTRODUCTION

With the ascending population and vehicles in modern cities, it is far from adequate to rely only on the ground space to fulfill the requirements of urban traffic life. Thus, the construction of underground tunnels is becoming the prior selection for urban transportation projects slowly and steadily. During the design and construction procedure of tunnels, the approaches to predict and control the deformation induced by tunnel excavation are of great significance. Under the condition of reasonable prediction of the ground movements, appropriate engineering measures could be applied to reduce the negative impact of the tunnel excavation on the above or underground structures.

Various methods have been proposed by different authors in scientific literature to estimate the ground movements along the horizontal distance and the extent of surface area influenced by the deformation phenomenon. The most commonly utilized empirical prediction method for the explanation of transverse surface displacements is proposed by Peck and Schmidt (Peck, 1969; Schmidt, 1969). It states that the ground movements caused by a circular tunnel with a certain radius are commonly characterized by a Gaussian distribution function. The function has been extended to the shallow tunnel in soft ground in the 1970s (Atkinson and Potts, 1977). For utilizing fewer input calculated parameters to gauge the measured data, assumptions of the soil constitutive model have been simplified to propose the two-dimensional analytical solution that predicts surface ground settlements (Sagaseta, 1987; Verruijt and Booker, 1996). The previous analytical solutions have been summarized by Pinto and extended to the three-dimensional distribution of surface ground settlements (Pinto and Whittle, 2014). The advanced numerical analyses offer a complete structure for modeling the processes involved in tunneling as well as the way they interact with other pre-existing structures, the approaches include artificial neural network, and the 3D finite element method (Suwansawat and Einstein, 2006; Migliazza et al., 2009).

Based on the abovementioned literature research, Peck's empirical method as well as improved Pinto's analytical method, and the finite element method realized by Plaxis 2D is selected to predict the ground movements induced by the excavation of Milan's underground extension as a case study. Detailed information regarding the project will be presented comprehensively in the following section.

2. PROJECT MATERIAL

In 2005, the extension segment of Line 1 of the Milan Metropolitan Subway was completed, which established a connection between the newly developed north metropolitan exhibition area and the city of Milan. Within the scope of the project, two tunnels were built with a ranging depth of 8 to 19 meters and an overall length of 2.1 kilometers. The plan view of the tunnel alignment is shown in Figure 1, the transverse sections selected for prediction calculation of three approaches are highlighted, which the depth of the tunnel spring line is 10.9m, 11.9m, 12.3m, and 13.4m along the direction of the excavation. The monitored surface ground settlements obtained from the control points will be compared with the prediction results for evaluation.

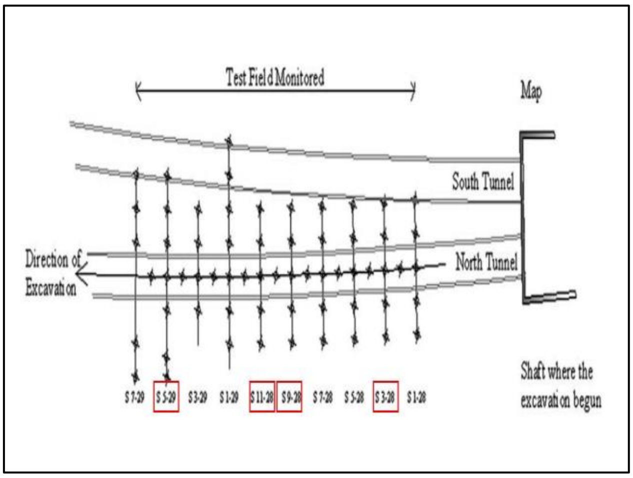

Figure 1 The top view of the excavated tunnels with the test field monitored control points

The earth pressure balance shield (EPB-S) machine whose cutter diameter is 6.54m was utilized in the construction processes, as well the outer diameter of the prefabricated concrete lining rings is 6.3m. The excavation works were carried out in strata consisting of dense sandy gravels originating from alluvial deposits, the basic soil properties from the site investigation are listed in Table 1. Considering the excavation of tunnels in sandy strata, polymer, and foam were applied to adjust the soil paste within the working chamber, and the gap between the sand mass and the lining was filled with grout injected from the ports surrounding the lining rings.

Table 1 Basic soil properties of dense sandy gravels

Physical soil properties	Values
Saturated unit weight	22.0 KN/m³
Dry unit weight	18.0 KN/m³
Friction angle	30°
Dilatancy angel	10°
Groundwater table	-10m
Mechanical soil properties	
Poisson's ratio	0.3
Elastic modulus (above GWT)	75 MPa
Elastic modulus (below GWT)	100 Mpa

3. METHODOLOGY

The filtered empirical method, improved analytical method, and finite element method accomplished by Plaxis 2D are briefly illustrated in this section, the distribution curves of vertical surface ground settlements predicted by the aforementioned approaches are exploited for comparison with the observed values obtained from the control points. The section also includes an introduction of the parameters utilized to assess the precision of the predicted outcomes estimated from various methods.

3.1 The Peck method

As the most recognized and utilized empirical formulation of ground movement prediction, the equation proposed by Peck and Schmidt reckons vertical settlements trough of the transverse section of the tunnel. the The maximum vertical ground settlement located at the tunnel centerline is calculated by:

$$S_{max} = \frac{\Delta V_L}{i_x * \sqrt{2\pi}} \quad (1)$$

And then the profile of the vertical ground surface settlements following the Gaussian distribution is expressed by:

$$S(x) = S_{max} * \exp\left(-\frac{x^2}{2i_x^2}\right) \quad (2)$$

S is vertical settlements at a certain horizontal distance, and x is the horizontal coordinate counted from the tunnel axis. ΔV_L is the ground loss at the tunnel, and it is calculated by the loss rate of soil and the cross-sectional area of the tunnel. i_x is the horizontal distance of the inflection point that affects the shape of the settlement trough, it is influenced by the excavated soil type and the depth of the tunnel axis. ΔV_L and i_x are determined by the empirical values summarized by former research (Han, 2006; Zhu and Li, 2017).

3.2 The Pinto method

Based on the formula proposed by Pinto and Whittle for shallow tunnels excavated in plastic soil, the simplified equation utilized in situ effective stress to predict the ground movements has been proposed by the study. The convergence strain and ovalization strain are calculated by:

$$u_\varepsilon = \frac{(p_0 - p_i)*R}{2G} \tag{3}$$

$$u_\delta = -\frac{q_0*R*(3-4\nu)}{2G} \tag{4}$$

p_0 and q_0 are the total volumetric stress and deviatoric stress at the tunnel spring line respectively, these two parameters are determined by the corresponding initial vertical total stress and horizontal total stress. The parameter *R* is the radius of the tunnel, and *G* and υ are the shear modulus and the Poisson's ratio of the excavated soil, respectively. p_i is the pressure induced by the grouting or the deformation of lining rings, the value equals the effective vertical stress of the tunnel crown according to the scientific literature (Bezuijen et al., 2002). The distribution of vertical surface ground settlements is described by the formula:

$$S(x) = \frac{2Z_t*R^{2\alpha-1}}{(x^2+Z_t^2)^\alpha} * \left\{u_\varepsilon - \frac{u_\delta*\left[(x^2+Z_t^2)^2-(3x^2-Z_t^2)(x^2+Z_t^2-R^2)\right]}{(x^2+Z_t^2)^2}\right\} \tag{5}$$

Z_t means the buried depth of the tunnel axis, and α is the coefficient of average dilation. It is calculated by:

$$\alpha = 1/(1-\sin\psi) \tag{6}$$

Ψ is the dilation angle of the soil.

3.3 Finite element method

Plaxis 2D was exploited to simulate the plain strain model of the cross-section of the excavated tunnel, the software is widely used in geotechnical engineering for performing finite element analysis of soil and rock structures, it also allows to define load and environmental conditions for analyzing the behavior of the structures. Within the study, due to the symmetry of soil structures, tunnel structures, and load conditions, the right-half region of the construction project has been modeled in the Plaxis 2D. The width of the model is 25m, as well as the depth, is 30m, and the depth of the groundwater table is -10m. Figure 2 shows the basic geometry of the simulated finite element model. The physical and mechanical properties of the soil obey the values from the investigation report, and the soil model utilized is the Mohr-Coulomb model, which represents the elastoplastic behavior of the soil.

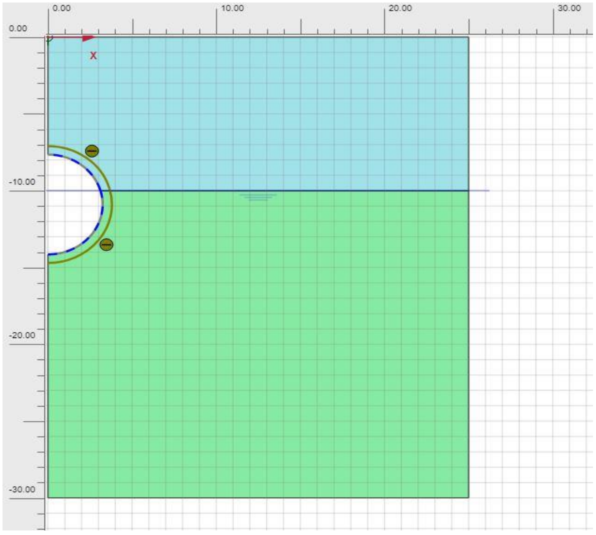

Figure 2 The finite element model established in Plaxis 2D

The outer diameter of the tunnel is 6.5m, and the lining rings are represented by the plate set in the software. In addition to the aforesaid information, the construction procedure has been simulated in four steps, including Tunneling, Contracting, Grouting, and Final lining. The final calculated results of the vertical deformation are displayed in Figure 3.

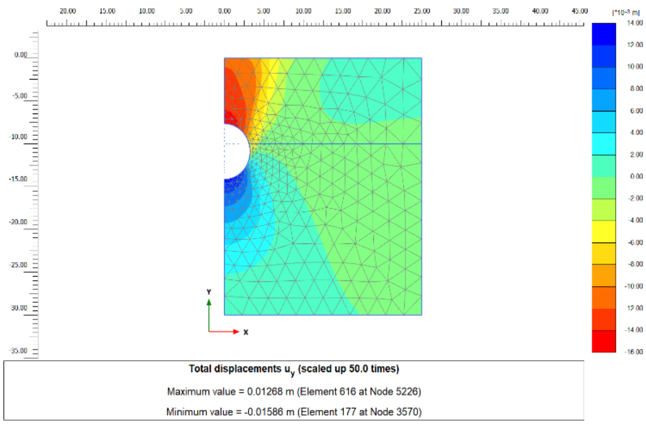

Figure 3 Total displacements of vertical direction calculated by FEM

3.4 Parameters for prediction evaluation

Three common parameters utilized for evaluating the estimation results are elaborated in the section, including mean absolute error (MAE), root mean squared error (RMSE), and symmetric mean absolute percentage error (SMAPE). The MAE is a non-negative value with smaller values indicating better predictive performance, it is calculated by:

$$MAE = \frac{1}{n} * \sum_{t=1}^{n} |F_t - A_t| \tag{7}$$

F_t is the predicted value and A_t is the true value.

The RMSE is similar to MAE, however, it penalizes larger errors than the MAE, it is defined by:

$$RMSE = \sqrt{\frac{1}{n} * \sum_{t=1}^{n}(F_t - A_t)^2} \quad (8)$$

SMAPE is similar to mean absolute percentage error, whereas it is less sensitie to the outliers, it is calculated by:

$$SMAPE = \frac{100}{n} * \sum_{t=1}^{n} \frac{|F_t - A_t|}{(|F_t| + |A_t|)/2} \quad (9)$$

The comparison results will be summarized in the section of the discussion.

4. DISCUSSION

After the calculation and summarization of the three separate prediction method, Figure 4 to 7 reflects the ground surface subsidence of measured data and predicted results with the variation of the horizontal distance in four transverse sections, in which the depth of the tunnel spring line ranges from 10.9m to 13.4m.

As shown in Figure 4, the shape of the calculated or simulated settlement trough is consistent with the ground truth, although the prediction settlement of the Peck method is apparently overestimated. The results estimated by the analytical method and finite element modeling are slightly underestimated and fairly close to each other. While the depth of the tunnel axis is getting deeper, the surface settlement trough is altered accordingly. From Figure 5, the measured settlement trough converted to flatter and wider than the settlements predicted by the Pinto analytical method are approaching the true value further. Apart from this, the FEM simulated values are still lower than the ground truth and the Peck empirical method possesses relatively good prediction results at a distance away from the centerline of the tunnel. In Figure 6 and Figure 7, the trend of the settlement trough remains consistent with the previous one, whereas, in these two cross-sections, the other two prediction methods overestimated the settlement values, only the prediction results of the FEM were best.

The overrating of the surface settlements calculated by the empirical Peck method results from two main reasons, one is that the experienced selected volume loss value of the Milanese dense sandy gravels is higher than the reality, and another cause is that the coefficient value of settlement trough width applied from empirical values is underrated. The situation is more common in the predicted results of empirical formulas, it is because the selection of several calculation parameters depends on the empirical values proposed by former researchers. The Pinto analytical method performs better prediction than the empirical method is caused of the more assumptions considered in the calculation, such as the effective stress of underlayers which results in the different deformation patterns in the analytical prediction.

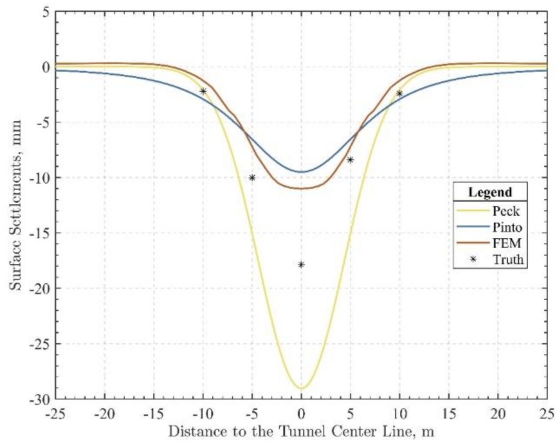

Figure 4 Settlements of the ground surface variate with the horizontal distance, the tunnel depth is 10.9m

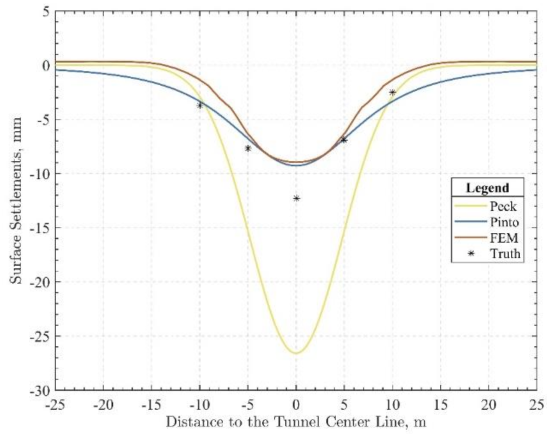

Figure 5 Settlements of the ground surface variate with the horizontal distance, the tunnel depth is 11.9m

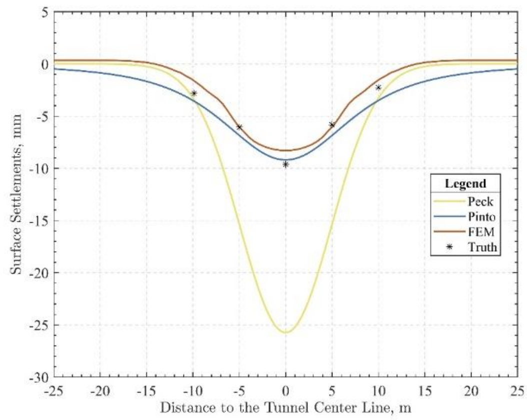

Figure 6 Settlements of the ground surface variate with the horizontal distance, the tunnel depth is 12.3m

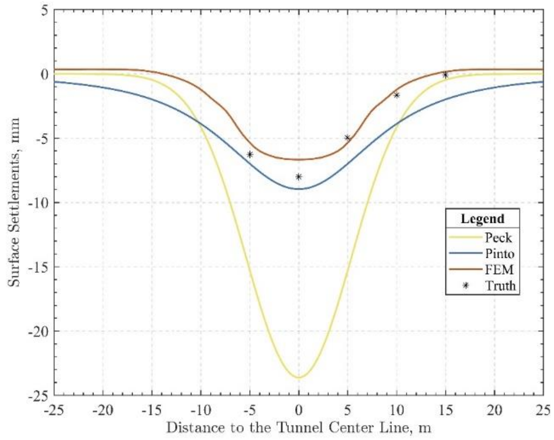

Figure 7 Settlements of the ground surface variate with the horizontal distance, the tunnel depth is 13.4m

As for the prediction performance of FEM, because of taking the physical and mechanical soil properties, the depth of the underground water table, the tunnel construction procedure, etc into account, the predicted results approximately approach the monitored subsidence. It outperformed the Pinto analytical method in a certain depth or horizontal distance.

In the specific horizontal distance is 0m, 5m, and 10m far away from the tunnel center line, the variations of vertical surface settlements with the depth of the tunnel axis are displayed from Figure 8 to Figure 10. The whole maximum vertical surface settlements shown in Figure 8 retain a similar tendency with the increasing of the tunnel axis depth, although the subsidence calculated by the empirical method has a distinct error compared with other results. Figure 9 shows that the values of vertical surface settlements predicted by the Pinto method and FEM are close to the ground truth, the variation trend of the Pinto method is contrary to the settlements variance of FEM. The dissimilar trend of the Pinto method is caused by the curve of the function image defined by the analytical solution.

Only the prediction results of FEM own a similar alternation with the true value which could be concluded from Figure 10, it proves that the ground vertical settlement predicted by FEM is more reliable when the horizontal distance is 1.5 times the diameter from the tunnel center line. The vertical ground surface settlements increases from 10.9m depth to 12.3m depth, which results from the escalating quantity of plastic points appeared at the certain location, the plastic deformation contributed to the increment of surface settlements.

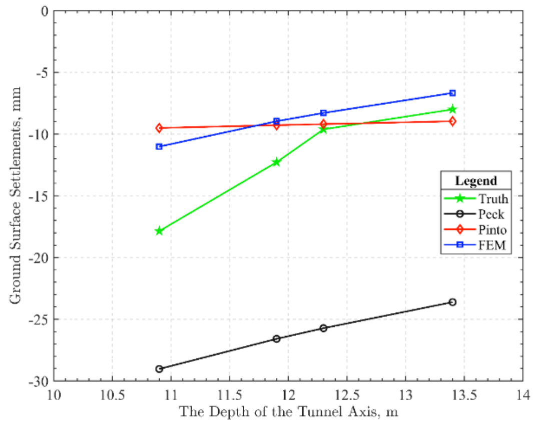

Figure 8 Variation of ground surface settlements with the depth of the tunnel axis of which the horizontal distance is 0m

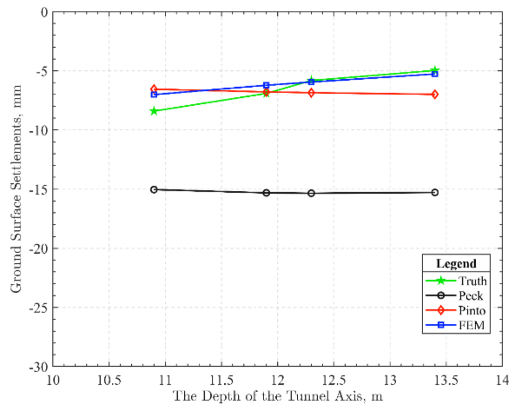

Figure 9 Variation of ground surface settlements with the depth of the tunnel axis of which the horizontal distance is 5m

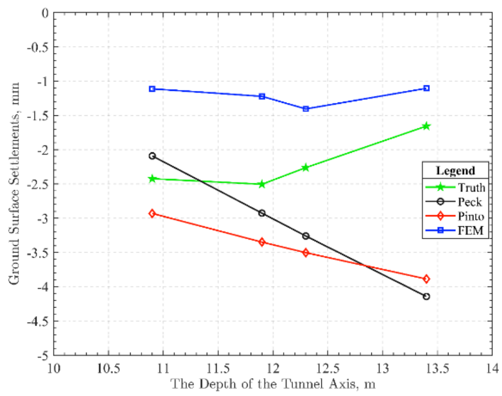

Figure 10 Variation of ground surface settlements with the depth of the tunnel axis of which the horizontal distance is 10m

The calculation results of several prediction evaluation parameters including mean absolute error, root mean squared error, and symmetric mean absolute percentage error are listed in Table 2. Compared with the prediction results of the empirical Peck method, either the analytical Pinto method or the FEM preserves lower values and better performance in MAE, RMSE, and SMAPE. The prediction effect of the FEM is slightly better than that of the analytical Pinto method when only the absolute error value is taken into account. However, when there are outliers in the observations, the analytical Pinto method retains better prediction performance. Therefore, both the analytical Pinto method and FEM are acceptable and accurate prediction approaches, and the optimal solution is determined according to the actual engineering or research demands.

Table 2 Evaluation of the settlement prediction results for utilized estimation approaches

Prediction Method	MAE (mm)	RMSE (mm)	SMAPE (%)
Peck Method	6.451	8.378	62.49
Pinto Method	1.606	2.394	33.682
FEM	1.508	2.122	44.404

5. CONCLUSION

The analysis of ground movements caused by the tunnel excavation has been implemented by the empirical Peck method, improved Pinto method, and finite element method. For the case study of the Melan metro line extension, the Peck method always overestimated the vertical ground surface settlements in several depths of the tunnel axis, which means the value estimated from the method can be used as a conservative reference value in engineering construction. The analytical Pinto method as well as the FEM significantly improved the performance of prediction and had great agreements with actually observed subsidence. It is proved that these two prediction methods are of great significance to the protection of nearby structures during the construction of underground infrastructure.

There are still several deficiencies in the analysis. The longitudinal vertical settlements are not focused on in the research, apart from this, the horizontal ground surface settlements in the transverse section and the longitudinal section of the tunnel should be paid more attention to in the following work.

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