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Initial Efficacy of a Fraction-Vocabulary Intervention for Students Experiencing Mathematics Difficulty in Grade 4

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This quasi-experimental study was designed to investigate the initial efficacy of a fraction-vocabulary intervention for Grade 4 students with mathematics difficulty (MD) and to explore its impact on relevant fraction competencies. Thirty-three students were assigned to either the intervention condition ($n = 16$) or a business-as-usual comparison condition ($n = 17$). The intervention occurred 3 sessions per week for 4 weeks, for a total of 11 sessions. Results indicated the main effect of the fraction-vocabulary intervention was significant for fraction-vocabulary posttest and one fraction competency—fraction arithmetic. That is, students with MD can successfully learn fraction vocabulary via a brief intervention, and improved fraction-vocabulary knowledge may positively affect their fraction competencies.

Understanding fractions is essential for more advanced mathematics and success in the American workforce (Booth & Newton, 2012; National Mathematics Advisory Panel [NMAP], 2008; Siegler et al., 2013). Nevertheless, fraction knowledge is one of the most complicated mathematical domains for students (Namkung et al., 2018; Schumacher et al., 2018). More alarming, limited understanding of fractions is especially prevalent among students with mathematics difficulty (MD), who also experience challenges with whole-number concepts and operations (Barbieri et al., 2020). In addition to difficulty with fractions, researchers have also reported students with MD exhibit difficulties in understanding and applying mathematics vocabulary (Forsyth & Powell, 2017; Lin et al., 2021). For instance, Forsyth and Powell (2017) revealed that fifth graders with the lowest computation scores demonstrated the lowest performance on a mathematics vocabulary measure. Similarly, Lin et al. (2021) determined the only common deficit shared by students with the lowest computation and word-problem solving scores was difficulty understanding mathematics vocabulary. Without adequate understanding of mathematics-vocabulary terms, students with MD may find it difficult to understand classroom instructions, engage in rich mathematical discussions, comprehend written text, or perform adequately on assessments.

Interventions to Improve Mathematics Vocabulary

US mathematics standards highlight the importance of using mathematics vocabulary as a key to learning mathematics (Common Core State Standards Initiative, 2010). Specifically, students should be able to express themselves to others, explain how to solve problems, and evaluate the mathematical reasoning of others using clear and precise vocabulary. Consistent with the focus on mathematics vocabulary in the mathematics standards, emerging research has demonstrated an association between mathematics vocabulary and mathematics learning (Peng & Lin, 2019; Powell & Nelson, 2017; Powell et al., 2017). For instance, Powell et al. (2017) noted a significant correlation between mathematics computation and mathematics vocabulary across third- and fifth-grade US students.

Despite significant evidence showing that understanding mathematics vocabulary is related to students' mathematics performance (Lin et al., 2021), there is limited research on interventions that target mathematics vocabulary as the primary focus. Instead, it is more common for mathematics interventions to incorporate mathematics-vocabulary instruction to support a different primary learning objective, such as enhancing proficiency in recognizing and comparing fractions, and solving word problems involving fractions. Some of these interventions—but not all—were conducted with students with MD (e.g., Hassinger-Das et al., 2015; Nelson & Kiss, 2021; Powell & Driver, 2015). Even though the results of these studies revealed significant gains in mathematics knowledge and mathematics vocabulary, it is not possible to differentiate the effects of the embedded mathematics-vocabulary instruction because the interventions engaged students in activities focusing on both enhancing mathematics knowledge and mathematics vocabulary. Therefore, it is necessary to investigate whether an intervention targeting mathematics vocabulary alone (i.e., without additional focus on mathematics knowledge and skills) would improve students' mathematical performance.

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Among the few interventions with a primary focus on mathematics-vocabulary instruction (Petersen-Brown et al., 2019; Purpura et al., 2017, 2021), Most were conducted in preschool children and used shared reading to promote mathematics vocabulary (Purpura et al., 2017, 2021). For example, Purpura et al. (2017) trained interventionists to read to children in small groups for eight weeks focusing on mathematics vocabulary in the classroom setting. This intervention led to significant gains in both mathematics vocabulary and early numeracy scores. Recently, Purpura et al. (2021) expanded this mathematics-vocabulary instruction to the home environment, where it was implemented by families, with similar positive effects. As such, these studies reveal that shared reading interventions can have positive impacts on children's mathematics vocabulary across different settings.

As the complexity of mathematics vocabulary increases by grade level (Powell et al., 2017), other strategies for teaching mathematics vocabulary may be necessary, above and beyond shared reading. One strategy used to teach vocabulary in reading and suggested for use in mathematics (Bruun et al., 2015) is a graphic organizer. One type of graphic organizer, the Frayer Model, visually assists students in selecting and organizing information regarding a central vocabulary (Frayer et al., 1969). The strategy consists of four sections: definitions, characteristics, examples, and nonexamples.

We utilized a modified version of the Frayer Model, called a Vocabulary Grid, that follows the framework established by Marin (2018). In this adapted version, we organized the materials into columns instead of quadrants and used a practice problem in place of the characteristics. We also included aspects of reading-vocabulary instruction deemed crucial in previous reviews (e.g., Jitendra et al., 2004), where explicit instruction of vocabulary and multiple exposures to vocabulary were noted as promising practices for teaching vocabulary terms. Specifically, when paired with explicit instruction, prior studies have demonstrated the effectiveness of using graphic organizers for improving vocabulary for students with learning disabilities and other high-incidence disabilities (Knight et al., 2013).

Possible Connection between Fraction-Vocabulary and Fraction Competencies

Increasing students' exposure to mathematics vocabulary may have a beneficial impact on their mathematical knowledge. Aside from detecting positive effects on mathematics vocabulary, Purpura et al. (2017, 2021) also revealed positive effects on early numeracy skills. This finding is similar to those of nonintervention studies in which researchers identified positive and strong correlations between mathematics vocabulary and performance on mathematics measures (Lin et al., 2021; Powell & Nelson, 2017; Powell et al., 2017). Notably, as the complexity of mathematics knowledge increased by grade level, mathematics vocabulary became accumulatively more complex and content-specific (Powell et al., 2017). In the current study, we examined whether improved fraction vocabulary led to improved fraction competencies (i.e., comparing fractions, fraction arithmetic, and fraction word problems).

According to Siegler et al. (2011), many US students lack a conceptual understanding of fractions, which, in turn, limits their capacity to solve problems with fractions. Given that mathematics vocabulary primarily represents conceptual knowledge stored in long-term memory (Schleppegrell, 2007), it follows that a strong understanding of fraction vocabulary may allow students to focus on the more cognitively demanding aspects of solving problems with fractions (Lai, 2011). A recent meta-analysis revealed fraction vocabulary was a critical factor in higher-order mathematics tasks (e.g., fractions) because it may serve as a medium that facilitates cognitive reasoning in mathematics learning (Lin et al., 2021). Thus, an intervention with a primary focus on fraction vocabulary may help enhance students' understanding of foundational fraction concepts, which, in turn, may benefit fraction competencies (e.g., comparing fractions, fraction arithmetic, and fraction word-problem solving).

The Present Study

To date, few interventions have prioritized mathematics-vocabulary instruction as their primary focus (Petersen-Brown et al., 2019; Purpura et al., 2017, 2021). However, none of these studies have specifically targeted students with MD. Furthermore, little is known about the impact of improving students' mathematics vocabulary (e.g., fraction vocabulary) at later elementary grades on their corresponding mathematics performance. To fill this gap in the literature, we investigated the following two research questions:

1. What is the initial efficacy of the fraction-vocabulary intervention implemented with students with MD?
2. Do the effects of fraction-vocabulary intervention transfer to other fraction competencies (i.e., comparing fractions, fraction arithmetic, and fraction word problems)?

METHOD

Participants

We received approval from our university Institutional Review Board (IRB) and our local school district to conduct research in schools. The participating school district served over 80,000 students. In 2021, the district reported 55% of students as Hispanic, 30.1% as White, 6.6% as Black, and 8.3% as belonging to another racial or ethnic category. Further, 31.2% of students qualified as English learners, and 13.3% received special education services. Overall, 51.9% of students qualified as economically disadvantaged.

We used a quasi-experimental study design with pretest, intervention, and posttest. Participants were recruited from students randomly assigned to a business-as-usual (BAU) comparison condition in a large-scale efficacy study funded by the Institute for Education Sciences (i.e., the parent study;

Powell & Doabler, 2020). The primary goal of the parent study was to test the efficacy of a word-problem intervention focused on single- and multistep word-problem solving for students with MD. Students within the parent study's BAU condition did not receive any intervention. To be eligible for the parent study, students had to be current fourth graders enrolled in a public elementary school within the participating school district and in a recruited teacher's classroom.

Of the 42 students assigned to the BAU condition of the parent study, we recruited and received caregiver consent and student assent from 33 BAU students. Caregivers consented to their child's participation, and students assented to participate. Due to time constraints, we assigned participant students based on the order in which we received consent and assent forms. Of the 33 students, 16 were assigned to the fraction-vocabulary intervention condition and 17 were assigned to the BAU condition. (Appendix A provides the demographics of the participants.)

Measures

Screening Measures

The parent study implemented two screeners to identify students experiencing MD. Specifically, students who scored at or below the 25th percentile on both screeners, a commonly used cutoff point in MD research, were identified as students experiencing MD (Nelson & Powell, 2018). On *Grade 4 Word-Problem Screener* (Powell & Berry, 2021), students solved 10 word problems. Eight were single-step word problems representing the total, difference, change, equal groups, and comparison schemas. Two problems involved multistep word problems (one with equal groups then total; the other with equal groups then equal groups). Cronbach's α was .86. The second screener was an abbreviated version of the *Intermediate 2 Mathematics Problem Solving* subtest from the Stanford Achievement Test (SAT)−10 (Pearson Inc., 2004). On this measure, students solved eight problems, all of which involved single-step word problems representing total, difference, change, and equal groups schemas. Cronbach's α was .81.

Examiners read each word problem aloud twice. Students had 1−2 minutes to solve each problem. We scored numerical responses as correct with a maximum of 18 points across the two measures.

Pretesting and Posttesting Measures

The testing battery consisted of four measures: *Fraction Vocabulary*, *Comparing Fractions*, *Fraction Arithmetic*, and *Fraction Word-Problem Solving*.

Fraction vocabulary. Fraction vocabulary was assessed with *Fraction Vocabulary* (Lin & Powell, 2022). For each of the 33 vocabulary terms, we developed three levels of questions (Haladyna & Rodriguez, 2013), categorized into recall, comprehension, and use in tasks. Recall questions required students to remember a specific term, comprehension questions assessed their understanding of the term (e.g., identifying an *improper fraction*), and use questions required students to apply their knowledge to generate a fraction (e.g., write a *unit fraction*). The 33 items were designed to have a roughly equal balance of recall, comprehension, and use in task response formats. (Please refer to Appendix B for a detailed overview of the terms present in the glossaries of Grades 3, 4, and 5 textbooks.) The score was the number of questions answered correctly within 15 minutes. Cronbach's α was .83.

Comparing fractions. We used *Comparing Fractions* (Schumacher et al., 2010) to assess magnitude understanding. Students had 8 minutes to write the greater than, less than, or equal sign between two fractions for 15 items. Students received one point for each correct answer. Items were as follows: $\frac{2}{6}$ and $\frac{4}{6}$; $\frac{1}{2}$ and $\frac{7}{10}$; $\frac{4}{12}$ and $\frac{1}{2}$; $\frac{3}{6}$ and $\frac{3}{8}$; $\frac{8}{12}$ and $\frac{1}{2}$; $\frac{1}{12}$ and $\frac{1}{5}$; $\frac{1}{2}$ and $\frac{5}{10}$; $\frac{4}{6}$ and $\frac{1}{2}$; $\frac{9}{10}$ and $\frac{5}{10}$; $\frac{1}{2}$ and $\frac{7}{8}$; $\frac{1}{2}$ and $\frac{3}{4}$; $\frac{3}{6}$ and $\frac{1}{2}$; $\frac{7}{8}$ and $\frac{7}{12}$; $\frac{1}{4}$ and $\frac{3}{4}$; and $\frac{1}{2}$ and $\frac{4}{8}$. The maximum score was 15. Cronbach's α was .67.

Fraction arithmetic. The *Fraction Arithmetic* (Lin, 2022a) involved four fraction addition and four subtraction items, which were administered within 2 minutes. Students received one point for each correct answer. Even though the Grade 4 curriculum included a focus on both like and unlike denominators, we only included like denominator problems because of our focus on students with MD. All problems involved fractions with the same denominators. Items were as follows: $\frac{1}{3} + \frac{4}{3}$; $1\frac{1}{2} + \frac{1}{2}$; $\frac{2}{5} + \frac{3}{5}$; $\frac{1}{4} + \frac{2}{4}$; $\frac{5}{6} - \frac{4}{6}$; $\frac{7}{6} - \frac{5}{6}$; $\frac{5}{8} - \frac{3}{8}$; $1\frac{3}{4} - \frac{1}{4}$. Cronbach's α was .93.

Fraction word-problem solving. On this measure, *Fraction Word-Problem Solving* (Lin, 2022b), students solved eight word problems (see Appendix C). The number sentences that corresponded to the word problems were identical to those on *Fraction Arithmetic*. Students received one point for each correct response for a maximum score of 8. Fraction word problems was administered for approximately 16 minutes and were read aloud to students. Cronbach's α was .82.

Intervention

The fraction-vocabulary intervention consisted of a total of 11 lessons, 10 instruction lessons plus a review lesson. Each instruction lesson introduced two to four closely related vocabulary terms, followed by a review lesson reviewing all the vocabulary terms. Eight students received intervention in pairs, while the other eight students received intervention individually. This tutoring occurred outside the general education classroom in a quiet space (e.g., school library, extra classroom) during students' regular mathematics class.

The fraction-vocabulary intervention consisted of 35 terms. In addition to 33 fraction-vocabulary terms from the fraction-vocabulary measure, we included *whole numbers* and *integers* to help students understand the meaning of *common fraction*. Table 1 provides a general outline of the fraction vocabulary introduced or reviewed in each lesson. The lesson briefly mentioned seven fraction-vocabulary terms, which were either covered in the definition of other fraction vocabulary (the meaning of *like fractions* was

TABLE 1
Fraction-Vocabulary Lesson Overview

Lessons	*Fraction-Vocabulary Terms*
1	*Whole*, *Equal Parts*, *Fraction*, *Whole Numbers*, *Integers*, *Common Fraction*
2	*Numerator*, *Denominator*
3	*Unit Fraction*, *One Half, One Fourth* (*On Fifth*, *One Twelfth*)
4	*Proper Fraction*, *Improper Fraction*, *Mixed Number*
5	*Benchmark Fraction*, *Fraction Greater Than 1*
6	*Factors*, *Greatest Common Factor*, *Simplest Form*
7	*Multiple*, *Least Common Multiple*, *Least Common Denominator*
8	*Like Denominators* (*Like Fractions*), *Unlike Denominators* (*Unlike Fractions*), *Equivalent Fractions*
9	*Decimal*, *Decimal Point*, *Decimal Place* (*Tenths Place*, *Hundredths Place*, *Thousandths Place*)
10	*Decimal Fractions*, *Equivalent Decimals*

Note. Fraction-vocabulary terms in parenthesis were briefly mentioned during the instruction of other fraction-vocabulary terms.

covered in the definition of *like denominators*) or concrete examples of primary fraction vocabulary (e.g., hundredths place is an example of decimal place).

Description of Intervention Activities

For each instruction lesson, intervention students participated in three activities: (1) Read and Match, (2) Vocabulary Grid Instruction, and (3) Vocabulary Log Review. For each review lesson, students participated in a Read and Match to review previously introduced fraction vocabulary. (See Figure 1 for a sample intervention worksheet.)

Read and match. To consolidate fraction-vocabulary knowledge, the tutor presented the student(s) with previously introduced fraction-vocabulary terms and definitions. Student(s) read the terms on the left side and definitions on the right side and drew a line to the correct definitions. Because Lesson 1 did not have a previous lesson, the Read and Match activity only occurred from Lessons 2 to 11. In the Read and Match activity, student(s) had approximately 2 minutes to match the definitions with the target terms. After the initial 2 minutes, the tutor provided immediate, corrective feedback on any noted errors.

Vocabulary grid instruction. The second activity consisted of tutor-led explicit instruction through the Vocabulary Grid, adapted from Marin (2018). During the delivery of instruction, tutors consistently observed the learners' understanding, closely monitored their progress, required frequent responses, and provided immediate feedback. The Vocabulary Grid included five columns: the term, definition, example, nonexample, and practice problem.

Step 1—rate the term. Before introducing the term's definition, the tutor asked the student(s) to rate, within 2 minutes, each term on a 1−4 rating scale (e.g., 1 = "No clue! I have never seen or heard this term before."; 2 = "I have seen or heard this term before."; 3 = "I have some idea of the term's meaning."; 4 = "I know this term, and I can define it.") based on their current knowledge of the term in a mathematical or another context.

Step 2—define the term. After the student(s) rated the term, the tutor introduced the definitions based on the student's familiarity with the term. In cases where the student had limited prior knowledge of the vocabulary term, the tutor provided a simple definition using language that was easy for the student to understand. However, if the student had some familiarity with the term, such as knowing its common English meaning (e.g., for *proper fraction*, *proper* means appropriate), the tutor helped the student make connections between this meaning and its mathematical definition. If the student had a great deal of understanding of the term, the tutor asked them to share their understanding and helped refine the student's mathematical definition. If the student knew the term and could define it, the tutor asked them to provide their definition and then provided corrective or affirmative feedback.

Step 3—reinforce meaning with examples and nonexamples. With the mathematical definition, the student(s) identified or generated examples and then nonexamples of each term in a mathematical context. These examples could be drawings and mathematical symbols.

Step 4—problem-based practices. This 2-minute activity allowed the student(s) to apply their vocabulary knowledge in context and motivated them to understand the learned fraction vocabulary better. Solving the problems required students to apply their understanding of the mathematical definition of the fraction vocabulary. (See Figure 1 for problems included in the intervention activity.)

Vocabulary log review. The Vocabulary Log Review involved a brief, timed review of the lesson contents. Student(s) had 2 minutes to write/say the newly introduced fraction vocabulary and their definitions and draw their representations on the Vocabulary Log Review. The student(s) performed the timed review autonomously and then received feedback from the tutor to correct any mistakes shown in their Vocabulary Log Review.

Intervention Training

The first author, along with three graduate research assistants, conducted all pretesting, tutoring, and posttesting. Each tutor held or was seeking a master's or doctoral degree in an education-related field. In early October of the school year, tutors participated in a 3-hour training, conducted by the first author to become familiar with the scripted lessons

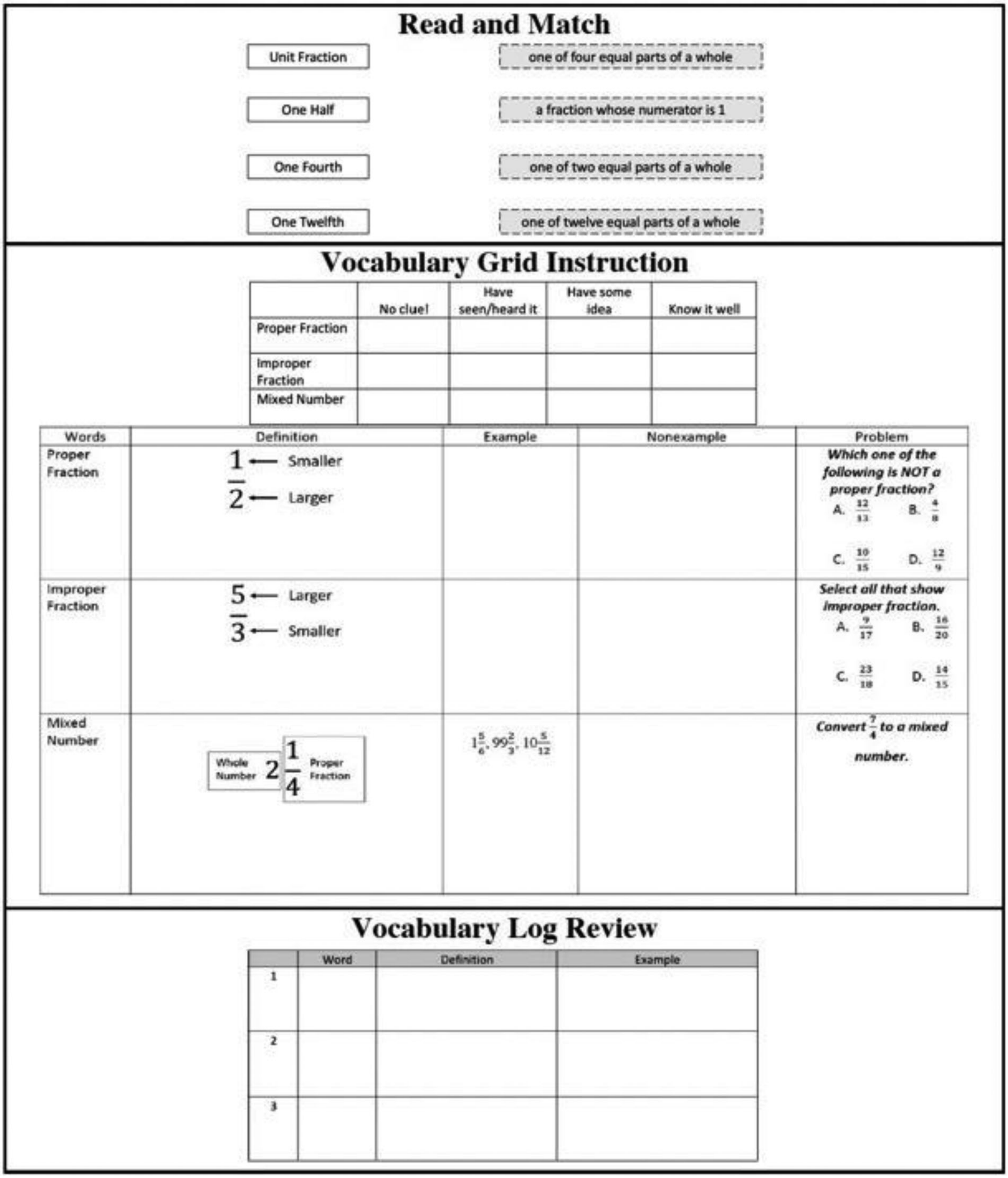

FIGURE 1 Sample intervention activities

and administration of the assessments. At the start of the training session, each tutor's fraction-vocabulary knowledge was also assessed. On average, tutors correctly answered 96.2% of the fraction-vocabulary items. Before their first tutoring session, tutors also completed reliability checks via pseudo-tutoring sessions with the first author and received corrective feedback. The first author continued to check in with the tutors virtually to discuss tutoring and resolve student behavior issues on a weekly basis. Each session was approximately 20 minutes.

Fidelity of Implementation

The lessons were scripted to ensure fidelity of implementation. Tutors audio-recorded every session. We randomly sampled >20% of sessions for each tutor and completed a checklist to identify whether the essential components were implemented as intended. For each lesson, a checklist of around 50 items needed to be marked off. (See Appendix D for an example fidelity checklist.) Fidelity averaged 95.6%, with a range of 86.3% to 100.0%. Tutors also tracked the number of sessions each student received in fraction-vocabulary intervention. On average, students completed 10.12 days of intervention (range 8 to 11; $SD = 1.11$). The duration of lesson was approximately 25 minutes, ranging from 22 to 29 minutes.

Business-as-Usual Comparison

Students in the BAU condition did not receive any intervention from our research team. Instead, they received mathematics instruction as would normally occur from their teachers. To gain a better understanding of what a typical Grade 4 mathematics class looked like for students in the BAU, we collected information from teachers via a survey about the fraction-vocabulary instruction students received. On average, teachers reported introducing the definitions of fraction-vocabulary terms a few times a week during the fraction instruction. Once a week, they taught fraction vocabulary utilizing examples, nonexamples, and visuals. Teachers reported using Frayer Models to teach fraction-vocabulary terms once a month.

Data Analysis

We utilized nonparametric tests because of the small sample size ($n < 25$) and nonnormally distributed variables. Prior to conducting model estimation, we conducted preliminary analyses to determine whether the variables were normally distributed. Preliminary inspection revealed significant kurtosis/skewness on the comparing fractions, fraction arithmetic, and fraction word-problem solving, thus violating the assumption that these variables were normally distributed. Only fraction vocabulary was normally distributed. Because the data for fraction competencies did not meet the assumptions required for parametric statistical tests, we used nonparametric tests to assess between-group differences.

We used the Mann–Whitney U test of significance to assess growth in fraction vocabulary and fraction competencies in intervention group and BAU group. We compared gains from pretest and posttest to be able to account for the initial level of student scores. We converted fraction competencies to difference scores (posttest scores – pretest scores). Finally, we calculated effect size using the following formula: $\frac{Z}{\sqrt{n}}$ for significant findings, where Z was the absolute standardized test statistic.

Results

Table 2 presents unadjusted pretest and posttest means, as well as adjusted posttest means, for students in the intervention and BAU conditions. At pretest, we identified no significant differences between intervention conditions on any of the four scores used in our analysis: fraction vocabulary ($U = 154$; $p = .533$), comparing fractions ($U = 148$; $p = .683$), fraction arithmetic ($U = 148$; $p = .709$), or fraction word-problem solving ($U = 156.5$; $p = .465$). In sum, intervention and BAU students demonstrated similar performance on preintervention fraction vocabulary and fraction competencies.

TABLE 2
Performance by Condition

	Intervention (n = *16*)		*Control* (n = *17*)	
Variable	M	SD/SE[a]	M	SD/SE[a]
Fraction vocabulary				
Pretest	8.81	3.43	8.59	4.76
Posttest	20.56	4.73	12.06	5.92
Adjusted post	20.50	1.25	12.12	1.21
Comparing fractions				
Pretest	8.44	2.68	8.53	3.57
Posttest	9.13	2.55	8.82	3.52
Adjusted posttest	9.16	0.42	8.79	0.41
Fraction arithmetic				
Pretest	4.62	2.53	3.88	3.50
Posttest	6.44	2.16	4.18	3.49
Adjusted posttest	6.16	0.49	4.44	0.47
Fraction word-problem solving				
Pretest	5.06	2.86	4.12	3.22
Posttest	6.00	2.13	4.35	3.57
Adjusted posttest	5.64	0.49	4.70	0.47

Notes: Adjusted posttest is posttest score adjusted for pretest score.
[a] For pre- and posttest, SD; for adjusted posttest, SE.

Fraction-Vocabulary Intervention Effects and Transfer Effects

To answer the first research question about the initial efficacy of the fraction-vocabulary intervention, we compared student gains on the fraction-vocabulary measure. Mann–Whitney U tests revealed significantly higher gains for students in the fraction-vocabulary intervention group than the BAU ($U = 242$; $p < .001$). The median fraction-vocabulary gain score was 12.50 for the intervention group compared to 3.00 for BAU students, which suggests the fraction-vocabulary intervention was effective. The effect size was 0.67.

To answer the second research question about the transfer effects of the fraction-vocabulary intervention to fraction competencies, we compared student gains on comparing fractions, fraction arithmetic, and fraction word-problem solving (see Table 3). Mann–Whitney U tests revealed significantly higher gains for the fraction-vocabulary intervention group compared to the BAU for fraction arithmetic ($U = 196$; $p = .031$). The median fraction-arithmetic gain score was 1.00 for the intervention group compared to 0.00 for the BAU, with an effect size of 0.39, which is a moderate to large

TABLE 3
Comparisons of Pretest and Gains on the Outcome Variables

Variable	*Intervention Median*	*BAU* Median	U	W	Z	p
Pretest						
Fraction vocabulary	9.0	7.0	154	290	0.652	.533
Compare fraction	8.0	8.0	148	284	0.445	.683
Fraction arithmetic	4.5	4.0	147	283	0.403	.709
Fraction word problems	5.5	5.0	157	292.5	0.754	.465
Gains						
Fraction vocabulary	12.5	3.0	242	378	3.829	.000
Compare fraction	0.0	0.0	158	294	0.847	.444
Fraction arithmetic	1.0	0.0	196	332	2.245	.031
Fraction word problems	0.0	0.0	160	295.5	0.883	.402

Note. BAU = business-as-usual; U = Mann–Whitney U; W = Wilcoxon W; Z = standardized test statistic.

effect. The mean fraction-arithmetic gain was 1.82 for the intervention group compared to 0.3 for the BAU. The comparisons on comparing fractions ($U = 158$; $p = .444$) and fraction word-problem solving ($U = 160$; $p = .402$) were not significant.

DISCUSSION

This study was conducted to determine the initial efficacy of a fraction-vocabulary intervention for Grade 4 students experiencing MD. Students who received the fraction-vocabulary intervention outperformed BAU students at posttest of the measures of fraction vocabulary and fraction arithmetic. Findings suggest, therefore, that a brief fraction-vocabulary intervention delivered through an adapted Frayer Model (i.e., a Vocabulary Grid) positively impacted students' fraction-vocabulary knowledge. We also learned that an intervention that concentrates mainly on enhancing fraction vocabulary can potentially enhance students' proficiency in fraction arithmetic. As this study was the first to develop and test a fraction-vocabulary intervention for Grade 4 students with MD, the study should be considered a pilot study, and results should be interpreted accordingly.

Initial Efficacy of the Fraction-Vocabulary Intervention

Students in the intervention group showed significant gains from pre- to posttest over the BAU group with a large effect size (0.67). Given the scarcity of intervention research that prioritizes mathematics vocabulary (Purpura et al., 2017, 2021), especially for students experiencing MD, future research needs to explore the most effective ways to teach mathematics vocabulary for students with and without MD. The present study provides a starting point to introduce, teach, and review fraction vocabulary to students with MD.

The finding about the promise of the fraction-vocabulary intervention suggests explicit instruction focused on building conceptual understanding of fraction vocabulary using the Vocabulary Grid can significantly improve fraction-vocabulary knowledge for students with MD. Besides using the Vocabulary Grid, the fraction-vocabulary intervention involved other components, such as the Read and Match activity that asked students to connect words and their definitions. However, given that the intervention incorporated several instructional components within the 11 lessons, it is difficult to identify which strategies were most effective for different types of fraction-vocabulary terms. That is, it is possible that the way tutors teach the term *least common denominator* needs to be different from the way they teach *decimal point*. Future research may consider empirically testing individual instructional strategies, such as using flashcards and mnemonics, for teaching different types of fraction-vocabulary terms.

Transfer to Fraction-Arithmetic Skill

Even though we identified a transfer effect only in fraction arithmetic, such an effect is crucial because students in the intervention group only received a brief intervention program (11 sessions) with a primary focus on fraction vocabulary. It is important to note that students in the intervention group did not participate in any fluency activities nor did the present intervention focus on fraction addition and subtraction. Therefore, the fraction-arithmetic outcome was not aligned with the fraction-vocabulary intervention but could be interpreted as a transfer measure. Given that classroom instruction allocated substantially more time to fraction arithmetic during the period in which the intervention occurred, the medium effect favoring intervention over BAU students on fraction arithmetic suggests that understanding fraction vocabulary may transfer to fraction arithmetic, at least for adding and subtracting fractions with like denominators. Compared to previous fraction intervention efforts that targeted a wide range of fraction competencies (e.g., Fuchs et al., 2014, 2017), such as fraction magnitude understanding, fluency building, and fraction addition and subtraction, the present intervention involved fewer sessions and only focused on fraction vocabulary.

Because core concepts and procedures in fractions and other disciplines are often learned through the use of vocabulary (Alexander-Shea, 2011), improved fraction-vocabulary knowledge may facilitate students' understanding of key concepts and procedures within fractions (Lin et al., 2021). In the process of acquiring fraction vocabulary, students form concepts about fractions. For example, students could successfully perform fraction addition and subtraction problems that involve fractions with like denominators (e.g., $\frac{1}{3} + \frac{4}{3}$) once they have better insight into the meanings of *fraction* (a number expressed as a quotient, in which a numerator is divided by a denominator) and *like denominators* (same denominators that means the whole is cut into the same number of pieces).

Understanding fraction vocabulary does not necessarily guarantee improved performance in comparing fractions and solving fraction word problems, at least when students participate in the brief faction vocabulary we implemented in this study. This finding is not terribly surprising because measures of comparing fractions and fraction word problems require a broader range of knowledge and skills than fraction arithmetic. For example, comparing fractions involves understanding the relative value of fractions, which includes comparing the size of numerators and denominators, finding equivalent fractions, and converting between mixed numbers and improper fractions. This requires a deeper understanding of fractions beyond understanding fraction vocabulary.

Limitations and Future Research

Despite the promising results, we note several limitations of this study. First, we used a quasi-experimental design. Even though intervention and BAU students demonstrated similar performance on preintervention fraction-vocabulary and fraction competencies, future research adopting a randomized-controlled trial is needed to eliminate confounding factors between the intervention and BAU conditions. Second, generalization of the results is limited by the small sample size. We intended for the study to be an exploratory pilot of a fraction-vocabulary intervention with students with MD. Despite the limited sample size, the size of the intervention effect was strong enough to attain statistical significance. It is recommended that future research use larger sample sizes to investigate the causal relation between mathematics vocabulary and mathematics knowledge among students experiencing MD. Third, we only used researcher-developed assessments to measure students' fraction competencies. In future studies, it would be essential to add standardized fraction assessments to determine whether fraction-vocabulary intervention positively affects fraction competencies. However, it is important to note that these measures were developed based on commercially available curricula, mathematics standards, and students' knowledge levels. Fourth, we collected data regarding the BAU group's regular fraction-vocabulary instruction through a survey instead of through direct observation.

In future research studies, researchers should build time into the intervention schedule to allow for students to participate in all 11 sessions for the fraction-vocabulary intervention. We were unable to conduct a complete tutoring program for each student due to student absences and schoolwide testing conflicts. Given the gaps in the tutoring schedule (i.e., some students were absent for three sessions), we hypothesize that more consistent implementation of tutoring lessons could improve the study's outcomes. Future research may also further investigate whether the efficacy of the intervention varies by group size, as noted by Doabler et al. (2019). Variance in group size relates to the number of opportunities each student has to respond and receive corrective feedback. In other words, researchers should investigate whether group size (1:2 [or greater] small-group tutoring versus 1:1 individual instruction) results in differences in fraction-vocabulary knowledge. Although the present intervention involved a mix of one-on-one and 1:2 small-group tutoring, Mann–Whitney U tests revealed a nonsignificant difference between 1:1 individual instruction and 1:2 small-group tutoring on fraction-vocabulary gains ($U = 24$; $p = .448$). Finally, future research should assess students' fraction-vocabulary knowledge several weeks and months after the immediate posttest to investigate whether the treatment effects would maintain or fade out over time.

Implications

Because students with MD benefitted from the fraction-vocabulary intervention delivered mainly through the use of a Vocabulary Grid (i.e., an adapted Frayer Model), we recommend using such a Vocabulary Grid that contain definitions, examples, nonexamples, and problem-based practice sections to teach mathematics vocabulary. Teachers could also employ variations of the present intervention components to support students' fraction-vocabulary understanding. For example, a brief problem-based practice problem (e.g., Put $\frac{2}{4}$ in *simplest form* is ___?) should be embedded into fraction instruction to ensure students demonstrate understanding of the target fraction-vocabulary term—*simplest form*. Students with MD also may benefit from explicit instruction on examples and nonexamples with practice opportunities to provide examples and nonexamples for each fraction-vocabulary term. When providing a nonexample, teachers could ask students to explain why it is a nonexample of the target fraction-vocabulary term and provide corrective feedback to enhance students' understanding.

Given that the present brief fraction-vocabulary intervention can potentially improve fraction arithmetic of fourth graders experiencing MD, future work may embed additional practice problems that are closely relevant to the target fraction-vocabulary term in the lesson to enhance the link between fraction-vocabulary and fraction competencies. For example, when introducing *like denominators* and *unlike denominators*, problems involving comparison or addition of fractions with like or unlike denominators (e.g., $\frac{7}{8}$and$\frac{7}{12}$; $\frac{1}{4} + \frac{3}{4}$) can facilitate students' understanding of the corresponding fraction concepts and procedures. Thus, a future study may compare the effectiveness of a fraction-vocabulary intervention condition and a fraction vocabulary with embedded fraction-competencies intervention condition on fraction-vocabulary and relevant fraction-competency outcomes.

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Supplementary Information

Additional supporting information may be found online in the Supporting Information section at the end of the article.

REFERENCES

Alexander-Shea, A. (2011). Redefining vocabulary: The new learning strategy for social studies. *The Social Studies*, *102*(3), 95–103. https://doi.org/10.1080/00377996.2010.509371

Barbieri, C. A., Rodrigues, J., Dyson, N., & Jordan, N. C. (2020). Improving fraction understanding in sixth graders with mathematics difficulties: Effects of a number line approach combined with cognitive learning strategies. *Journal of Educational Psychology*, *112*(3), 628–648. https://doi.org/10.1037/edu0000384

Booth, J. L., & Newton, K. J. (2012). Fractions: Could they really be the gatekeeper's doorman? *Contemporary Educational Psychology*, *37*(4), 247–253. https://doi.org/10.1016/j.cedpsych.2012.07.001

Bruun, F., Diaz, J. M., & Dykes, V. J. (2015). The language of mathematics. *Teaching Children Mathematics*, *21*(9), 530–536. https://doi.org/10.5951/teacchilmath.21.9.0530

Common Core State Standards Initiative. (2010). Common core state standards for mathematics. National Governors Association Center for Best Practices, Council of Chief State School Officers.

Doabler, C. T., Clarke, B., Kosty, D., Kurtz-Nelson, E., Fien, H., Smolkowski, K., & Baker, S. K. (2019). Examining the impact of group size on the treatment intensity of a tier 2 mathematics intervention within a systematic framework of replication. *Journal of Learning Disabilities*, *52*(2), 168–180. https://doi.org/10.1177/2F0022219418789376

Forsyth, S. R., & Powell, S. R. (2017). Differences in the mathematics-vocabulary knowledge of fifth-grade students with and without learning difficulties. *Learning Disabilities Research & Practice*, *32*(4), 231–245. https://doi.org/10.1111/ldrp.12144

Frayer, D., Frederick, W., & Klausmeier, H. (1969). *A schema for testing the level of cognitive mastery*. Wisconsin Center for Education Research.

Fuchs, L. S., Malone, A. S., Schumacher, R. F., Namkung, J., & Wang, A. (2017). Fraction intervention for students with mathematics difficulties: Lessons learned from five randomized controlled trials. *Journal of Learning Disabilities*, *50*(6), 631–639. https://doi.org/10.1177/2F0022219416677249

Fuchs, L. S., Schumacher, R. F., Sterba, S. K., Long, J., Namkung, J., Malone, A., Hamlett, C. L., Jordan, N. C., Gersten, R., Siegler, R. S., & Changas, P. (2014). Does working memory moderate the effects of fraction intervention? An aptitude-treatment interaction. *Journal of Educational Psychology*, *106*(2), 499–514. https://doi.org/10.1037/a0034341

Haladyna, T. M., & Rodriguez, M. C. (2013). *Developing and validating test items*. Routledge.

Hassinger-Das, B., Jordan, N. C., & Dyson, N. (2015). Reading stories to learn math: Mathematics vocabulary instruction for children with early numeracy difficulties. *The Elementary School Journal*, *116*(2), 242–264. https://doi.org/10.1086/683986

Jitendra, A. K., Edwards, L. L., Sacks, G., & Jacobson, L. A. (2004). What research says about vocabulary instruction for students with learning disabilities. *Exceptional Children*, *70*(3), 299–322. https://doi.org/10.1177/2F001440290407000303

Knight, V. F., Spooner, F., Browder, D. M., Smith, B. R., & Wood, C. L. (2013). Using systematic instruction and graphic organizers to teach science concepts to students with autism spectrum disorders and intellectual disability. *Focus on Autism and Other Developmental Disabilities*, *28*(2), 115–126. https://doi.org/10.1177/1088357612475301

Lai, E. R. (2011). Critical thinking: A literature review. Pearson's Research Reports. https://images.pearsonassessments.com/images/tmrs/CriticalThinkingReviewFINAL.pdf

Lin, X. (2022a). Fraction arithmetic. Available from X. Lin, Avenida da Universidade, Taipa, Macau.

Lin, X. (2022b).Fraction word-problem solving. Available from X. Lin, Avenida da Universidade, Taipa, Macau.

Lin, X., & Peng, P., & Luo, H. J. (2021). The deficit profile of elementary students with computational difficulties versus word problem-solving difficulties. *Learning Disability Quarterly*, *44*(2), 110–122. https://doi.org/10.1177/0731948719865499

Lin, X., Peng, P., & Zeng, J. G. (2021). Understanding the relation between mathematics vocabulary and mathematics performance: A meta-analysis. *The Elementary School Journal*, *121*(3), 504–540. https://doi.org/10.1086/712504

Lin, X., & Powell, S. (2022). Fraction vocabulary. Available from X. Lin, Avenida da Universidade, Taipa, Macau.

Marin, K. A. (2018). Routinizing mathematics vocabulary: The vocab grid. *Mathematics Teaching in the Middle School*, *23*(7), 395–398. https://doi.org/10.5951/mathteacmiddscho.23.7.0395

Namkung, J. M., Fuchs, L. S., & Kozilo, N. (2018). Does initial learning about the meaning of fractions present similar challenges for students with and without adequate whole-number skill? *Learning and Individual Differences*, *61*(1), 151–157. https://doi.org/10.1016/j.lindif.2017.11.018

National Mathematics Advisory Panel. (2008). Foundations for success: The final report of the National Mathematics Advisory Panel. U.S. Department of Education. http://www.ed.gov/about/bdscomm/list/mathpanel/reprot/final-report.pdf.

Nelson, G., & Kiss, A. J. (2021). Effects of a first-grade mathematics vocabulary intervention: A pilot study for students with mathematics difficulty. *Learning Disabilities Research & Practice*, *36*(2), 167–178. https://doi.org/10.1111/ldrp.12242

Nelson, G., & Powell, S. R. (2018). A systematic review of longitudinal studies of mathematics difficulty. *Journal of Learning Disabilities*, *51*(6), 523–539. https://doi.org/10.1177/0022219417714773

Pearson Inc. (2004). Stanford Achievement Test, Tenth Edition (SAT-10).

Peng, P., & Lin, X. (2019). The relation between mathematics vocabulary and mathematics performance among fourth graders. *Learning and Individual Differences*, *69*(1), 11–21. https://doi.org/10.1016/j.lindif.2018.11.006

Petersen-Brown, S., Lundberg, A. R., Ray, J. E., Dela Paz, I. N., Riss, C. L., & Panahon, C. J. (2019). Applying spaced practice in the schools to teach math vocabulary. *Psychology in the Schools*, *56*(6), 977–991. https://doi.org/10.1002/pits.22248

Powell, S. R., & Berry, K. A. (2021). Grade 4 Word-Problem Screener. Available from S. R. Powell, 1912 Speedway, D5300, Austin, TX 78712.

Powell, S. R., & Doabler, C. (2020–2024). The role of algebraic reasoning within additive and multiplicative multi-step problem solving for students with mathematics difficulty (R324A200176). U.S. Department of Education, National Center for Special Education Research, Institute of Education Sciences. https://ies.ed.gov/funding/grantsearch/details.asp?ID=4438

Powell, S. R., & Driver, M. K. (2015). The influence of mathematics vocabulary instruction embedded within addition tutoring for first-grade students with mathematics difficulty. *Learning Disability Quarterly*, *38*(4), 221–233. https://doi.org/10.1177/2F0731948714564574

Powell, S. R., Driver, M. K., Roberts, G., & Fall, A.-M. (2017). An analysis of the mathematics vocabulary knowledge of third- and fifth-grade students: Connections to general vocabulary and mathematics computation. *Learning and Individual Differences*, *57*(5), 22–32. https://doi.org/10.1016/j.lindif.2017.05.011

Powell, S. R., & Nelson, G. (2017). An investigation of the mathematics-vocabulary knowledge of first-grade students. *The Elementary School Journal*, *117*(4), 664–686. https://doi.org/10.1086/691604

Purpura, D. J., Napoli, A. R., Wehrspann, E. A., & Gold, Z. S. (2017). Causal connections between mathematical language and mathematical knowledge: A dialogic reading intervention. *Journal of Research on Educational Effectiveness*, *10*(1), 116–137. https://doi.org/10.1080/19345747.2016.1204639

Purpura, D. J., Schmitt, S. A., Napoli, A. R., Dobbs-Oates, J., King, Y. A., Hornburg, C. B., Westerberg, L., Borriello, G. A., Bryant,

L. M., Anaya, L. Y., Kung, M., Litkowski, E., Lin, J., & Rolan, E. (2021). Engaging caregivers and children in picture books: A family-implemented mathematical language intervention. *Journal of Educational Psychology*, *113*(7), 1338–1353. https://doi.org/10.1037/edu0000662

Schleppegrell, M. J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly*, *23*(2), 139–159. https://doi.org/10.1080/10573560601158461

Schumacher, R. F., Jayanthi, M., Gersten, R., Dimino, J., Spallone, S., & Haymond, K. S. (2018). Using the number line to promote understanding of fractions for struggling fifth graders: A formative pilot study. *Learning Disabilities Research and Practice*, *33*(4), 192–206. https://doi.org/10.1111/ldrp.12169

Schumacher, R. F., Namkung, J. M., & Fuchs, L. S. (2010). 2010 Fraction Battery. L. S. Fuchs, 228 Peabody, Vanderbilt University, Nashville, TN 37203.

Siegler, R. S., Thompson, C. A., & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Cognitive Psychology*, *62*(4), 273–296. https://doi.org/10.1016/j.cogpsych.2011.03.001

Siegler, R. S., Fazio, L. K., Bailey, D. H., & Zhou, X. (2013). Fractions: The new frontier for theories of numerical development. *Trends in Cognitive Sciences*, *17*(1), 13–19. https://doi.org/10.1016/j.tics.2012.11.004

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