Education and Information Technologies
https://doi.org/10.1007/s10639-022-11429-2

Peer tutoring models in collaborative learning of mathematical problem solving and their effect on group achievement

Yangyang Li[1,2] · Chunlian Jiang[3] · Zengzhao Chen[1,2] · Jing Fang[1,2] · Chenyang Wang[1,2] · Xiuling He[1,2]

Received: 3 August 2022 / Accepted: 24 October 2022

Abstract
This study was conducted to examine peer tutoring models in collaborative learning of mathematical problem solving (MPS) in flipped classrooms and their effect on group achievement. Quantitative data collected include 32 videos of eight groups of students in four MPS periods that were designed based on a simplified version of Polya's four-stage model, and their worksheets completed in each period. The video data were coded based on a framework with low-level and high-level cognitive behaviors as well as management behaviors and were analyzed using lag sequence analysis from which three types of peer tutoring models were determined: organization-oriented, cognitive-supported, and cognitive-guided. ANOVA analysis showed that the groups with cognitive-guided tutors performed significantly better than the groups guided by cognitive-supported and organization-oriented tutors, between which the performance difference did not reach a significant level. The results of this study suggest that training provided to peer tutors should focus more on how to stimulate high-level cognitive thinking skills than on organization skills.

Keywords Collaborative Learning · Flipped Classroom · Lag Sequence Analysis · Mathematical Problem Solving · Peer Tutoring Models

1 Introduction

The flipped classroom, a comparatively new teaching model, provides an interactive and flexible learning environment for developing students' problem-solving skills (Hwang & Chen, 2019). Indeed, collaborative learning is one of the appropriate teaching methods for the in-class portion of flipped instruction (Bishop & Verleger,

✉ Xiuling He
xlhe@ccnu.edu.cn

Extended author information available on the last page of the article

Published online: 19 November 2022

2013; Foldnes, 2016; Hwang & Chen, 2019). In a flipped classroom, students can have more face-to-face interaction, collaboration, and discussion with their peers, which allows them to form a deeper understanding of the content (Hwang & Chen, 2019) and to improve their performance (Foldnes, 2016). Peer tutoring or peer mentoring behaviors are often studied for their impact on the effectiveness of collaborative learning activities (Berghmans et al., 2013; Chu et al., 2017; De Smet et al., 2008; Nawaz & Rehman, 2017). A peer tutor is one who takes responsibility for organizing collaborative learning activities and often plays a leadership role in a group (Goodrich, 2018). They might, for example, actively promote the group's development through asking questions, providing suggestions, supporting and helping team members, and promoting problem-solving procedures (Goodrich, 2018). The positive effects of peer tutors have been found in a variety of subjects including mathematics (Chu et al., 2017) and, in particular, mathematical problem solving (MPS; Zulaiha et al., 2020). Chu et al. (2017) proposed a formative peer-tutoring approach in which a student whose performance is better on one learning activity can take the leading role in the next learning activity. They found that this new approach not only significantly improved students' achievement, but also reduced their cognitive load while discussing, providing hints, and guiding the group to solve the mathematics problems. However, it was also revealed that it is a big challenge for peer tutors to lead their groups in collaborative learning (Chu et al., 2017). Therefore, further research is needed to investigate peer tutoring models. This study examined an instructional model for a flipped classroom with group collaborative learning of MPS. The goal was to investigate peer tutoring models and to explore their impact on the effectiveness of group learning. In particular, this study explored what roles played by peer tutors are more important to the group's performance in collaborative learning of MPS. The findings will therefore inform the training provided to peer tutors so that they can play their roles more effectively in collaborative learning.

2 Literature review

In order to provide an understanding of the context on the peer tutors' role in collaborative learning of MPS in a flipped classroom, the literature review will cover the following four aspects: flipped classrooms, collaborative learning, peer tutoring models, and MPS.

2.1 Flipped classroom

The flipped classroom is defined as *"a new pedagogical method, which employs asynchronous video lectures and practice problems as homework, and active, group-based problem-solving activities in the classroom"* (Bishop & Verleger, 2013, p.1). Traditionally students come to class to listen to teachers' lectures and do the practice at home; these two activities are flipped in the new approach.

The flipped classroom has been promoted in education in recent years because it can increase student engagement (Wang & Zhu, 2019) and motivation to learn (Zheng et al., 2020). It can also enhance teacher-student interaction (Sun & Wu, 2016) and student–student interaction (Galway et al., 2014) in classes. It enhances learners' problem-solving potential (Hwang & Chen, 2019), promotes positive emotions (Jdaitawi, 2020), and helps develop students' computational thinking (Gong et al., 2020) and creativity (Al-Zahrani, 2015). In addition, it enables collaborative learning (Erbil, 2020), personalized learning, and flexible learning (González-Gómez et al., 2016; Nguyen et al., 2016). It is also widely used in mathematics instruction (Bhagat et al., 2016; Lai & Hwang, 2016; Sahin et al., 2015). For example, Bhagat et al. (2016) compared the flipped classroom and traditional teaching methods to teach trigonometry. They found that the students who used the flipped classroom method performed better, particularly the low-achievers.

The flipped classroom has become an innovative teaching model in China's new wave of educational reform (Wei et al., 2020). Therefore, this study adopts the flipped classroom teaching model to teach MPS. In the use of the flipped classroom, there are normally three stages: before, in, and after class. Before class, students watch a micro video about MPS and complete a worksheet independently. The video and the worksheet are designed by the mathematics teacher. In class, groups are formed to discuss their solutions to the questions in the worksheet through collaborative learning activities, including individual presentation and explanation, group discussion, summarization, evaluation, and reflection. After class, students revise their own solutions to the questions in the worksheet and reflect on and summarize what they have learned. In the whole process, the teacher mainly provides guidance, assistance, and other support for students and groups.

2.2 Collaborative learning

Collaborative learning methods are often used in the in-class session of the flipped classroom (Bergmann & Sams, 2012; McLaughlin et al., 2013; Tucker, 2012). Research has indicated that the use of collaborative learning approaches in flipped classrooms has a positive impact on learners' academic achievement (Foldnes, 2016; Shi et al., 2020), attitudes (Qiang, 2018), and behaviors (Munir et al., 2018). It can also improve the effectiveness of the teaching and learning process (Erbil, 2020).

Collaborative learning can enhance students' learning only when teachers carefully plan collaborative learning activities and help students use collaborative learning skills appropriately in the classroom (Johnson et al., 2000). To guide students' collaborative learning activities, a four-phase model proposed by Tong (2017) based on the collaborative knowledge construction process model was adapted in the current study. In particular, the four phases are: (a) Information-sharing phase, (b) Conflict-negotiating phase, (c) Consensus-reaching phase, and (d) Reflecting phase. This study combined the third and the fourth phases into one as the consensus-reaching phase, and a preparation phase was added at the beginning for students to upload their worksheets to the cloud. How the collaborative learning was organized into the four phases is described in more detail in the method section.

To be effectively involved in collaborative learning, students also need to learn the rules for collaboration (Munir et al., 2018). Therefore, the researcher and the mathematics teacher worked together to develop a collaboration and communication guidelines including thinking independently, speaking softly, learning to listen and to provide help, daring to ask questions and doubting authority, learning how to encourage and praise, and so forth. Before the experiment began, the researcher provided training to the students so that they had developed the basic collaboration and communication skills.

2.3 Peer tutoring models

A peer tutor is one who is selected or assigned as the leader of a group for the implementation of collaborative learning. The interaction patterns led by the peer tutors are directly related to the groups' engagement level and achievement (Berghmans et al., 2013; Madaio et al., 2017). Therefore, it is necessary to provide training to the peer tutors to improve their ability to play a leading role. The training is intended to help them to develop an understanding of their roles as peer tutors, to improve their communication skills on how to develop and maintain good relationships with the team members, and to improve their ability to manage time, and so forth (Colvin & Ashman, 2010).

Researchers have used a variety of approaches to explore peer tutoring behaviors or models (Berghmans et al., 2013; Capstick & Fleming, 2004; De Smet et al., 2008; Sun et al., 2021). For example, De Smet et al. (2008) used cluster analysis to classify peer tutors into three types: informers, motivators, and knowledge constructors. Informers tend to spread and explain knowledge while motivators focus on encouraging and motivating peers to participate. Knowledge constructors play the roles of both informer and motivator when necessary, which means that they are able to pay more attention to inspiring and promoting peers' learning, and give peers' greater responsibilities. Sun et al. (2021) used lag sequence analysis (LSA) to analyze the differences in knowledge-construction behaviors between high and low-performing groups in a mobile learning environment. They found that all the learners moved smoothly from lower to higher phases of knowledge construction and the high-performing group attained a deeper level of knowledge construction through the negotiation of meaning. In our study, we used LSA methods to analyze and visualize the tutoring behaviors of peer tutors in collaborative learning, to identify the potential tutoring models of different groups imbedded in behavioral data, and finally to examine the effects of different peer tutoring models on students' MPS performance.

2.4 Mathematical problem solving (MPS)

MPS has become one of the main objectives of mathematics education in schools in different countries (Curriculum Planning and Development Division [CPDD], 2019, 2020; National Council of Teachers of Mathematics [NCTM], 2000). Items related to MPS account for 75% of the mathematical domains in the Program for

International Student Assessment (PISA) 2021 (Organization for Economic Co-operation and Development [OECD], 2018).

Polya's four-stage model was widely used for teaching and research in MPS (Hembree, 1992) for its importance in helping one tackle non-routine tasks systematically and effectively (CPDD, 2019). This study used a modified version of Polya's mathematical problem-solving model as a scaffolding tool (Fig. 1) with Polya's second and third steps combined. In particular, the worksheets that we designed for use with the students included the following three steps: (a) read and understand the problem, (b) analyze and solve the problem, and (c) look back and reflect. In the first step, the students read the problem to find out (a) what are the givens in the problem and the relationship among the givens, and (b) what is the unknown. The second step includes Polya's second and third steps. In Chinese mathematics education, Polya's second step, "Devise a plan," has been renamed as "analyzing the problem," which normally refers to find the way from the initial state to the goal state (Anderson, 1993). This step was facilitated with a variety of representation tools including line models, which are similar to the model-drawing approach used in Singapore primary mathematics (Jiang & Chua, 2010). The output of Polya's "carry out the plan" step is the problem solution. In this study, the presentation was taken as an unseparated part of the solutions. In the last step, solution approaches together with the related mathematical knowledge and skills were summarized. In the before-class stage, the students tried to solve the problems independently following the steps. In the in-class stage, the students worked in groups to solve the

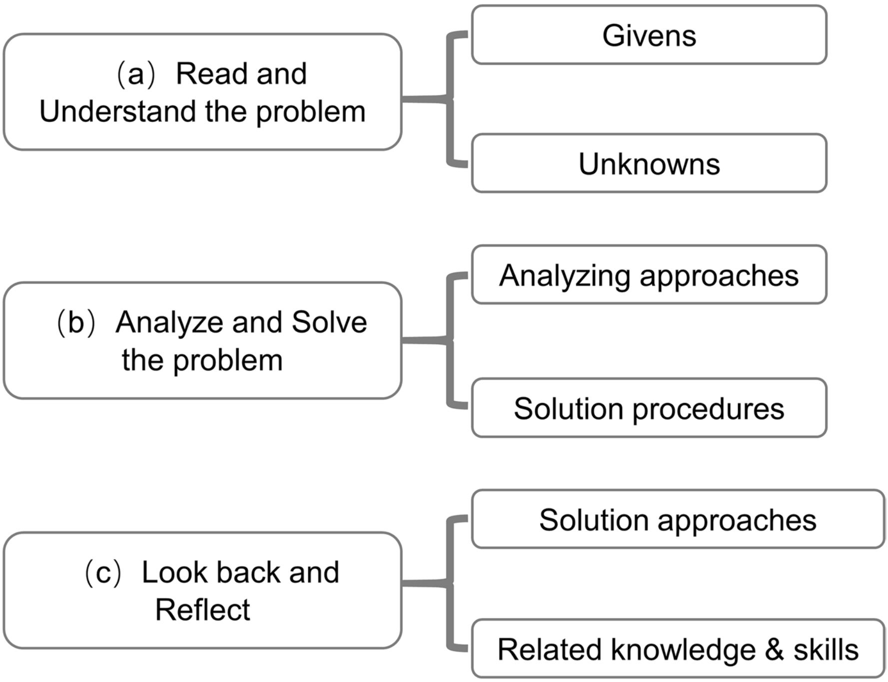


Fig. 1 Modified Polya's Mathematical Problem-solving Model

problems collaboratively. As an example, a worksheet used in one of the four periods was included in Appendix's Figure 6.

In summary, this study was conducted to investigate peer tutoring models in the in-class discussion stage of the flipped classroom of MPS guided by a modified Polya's model. Specifically, this study was intended to address the following research questions:

(a) What are the characteristics of peer tutoring models in collaborative MPS learning in the flipped classroom?
(b) How do different peer tutoring models affect groups' MPS achievement?

3 Methodology

3.1 Research context

The study was conducted in two fifth-grade mathematics classes taught by the same teacher in a primary school in Xiamen, China. The participants included 95 students (45 boys and 50 girls) whose average age is ten years. Ethical permission was obtained from the school before the study.

Instruction was carried out in a smart classroom which was linked to a cloud learning platform developed by the authors' team. The platform includes three clients: the teacher client, the student client, and the group client, which allows interaction and collaboration between the teacher and the students, between two students, and among students within a group. In addition, the following items were presented in the classroom: (a) five interactive screens, including one for the teacher and one for each group with a total of four; and (b) one tablet for each student so that they could access online learning resources and upload their work to the cloud.

This study took place over five weeks. In the first week, a 40-min technical training session was conducted for the students to familiarize them with the smart classroom environment and the methods of collaboration and communication with peers. The students in each class were then grouped into four groups by their mathematics teacher based on their performance on the last mathematics test they took before the study to ensure that each group had a spread of different performance levels. Meanwhile, we performed an ANOVA on the eight groups and found no significant differences among them ($F(7, 87) = 2.249$, $p = 0.111$). Each group consisted of 11–12 students, one of whom was assigned to be a peer tutor by the teacher. The peer tutor did not only perform well in the mathematics test concerned, but also was able to communicate well with the group member. The eight groups were labeled G1, G2, …, G8, and the corresponding peer tutors were labeled T1, T2, …, T8. As mentioned above, we also provided training for peer tutors to help them understand their role and the importance of communication skills as leaders.

Flipped classroom instruction was carried out from the second to the fifth weeks with one MPS period (80 min) on two tasks each week. The activities before and after the classes were as described above in the review of literature on flipped classrooms. Here, we shall describe the in-class activities in detail according to the four phases: (a) *Preparation phase*. Students sat around the group screen and took pictures of their worksheets through the student clients and uploaded them to the group client to share information. (b) *Information-sharing phase*. Group members explained and shared their solutions standing in front of the group screen one by one, and they could use annotation to highlight the main points in their solution through the group client. At the same time, they tried to answer questions raised by their own group members. (c) *Conflict-negotiating phase*. Students compared their solutions as a group, identified conflicting or different ideas, and made arguments through questioning, evaluating, dialogue, and so forth to negotiate meaning or co-construct knowledge. (d) *Consensus-reaching phase*. Students tried to synthesize the group's views to propose final suggestions and solutions to build consensus. Finally, the group solutions were submitted to the teacher client to end the class. In the last three phases, the group worked together through the group client. Throughout the in-class period, peer tutors were responsible for the smooth running of the collaborative learning activities, which is the main focus of the current study.

3.2 Data sources

To answer the two research questions, two sets of quantitative data were collected: (a) 32 videos of the collaborative learning process with four videos for each of the eight groups. The videos recorded the entire learning process of each group as well as the communication behaviors of both peer tutors and tutees. (b) The participants' MPS performance, which consisted of the scores obtained on the revised worksheet after each period. The marking was done by the teacher. For one task, a maximum of 10 marks, 30 marks, and 10 marks were given to the first, the second, and the last step of the modified Polya's stages, respectively. Therefore, the total marks for one task was 50 marks, and the total for each MPS period was 100 marks. The scores in each period and the mean score in the four periods were used to determine whether there were significant differences among students under different tutoring models.

3.3 Data coding

This study used a modified coding scheme of learner interaction behaviors developed by Wang et al. (2016). Wang et al. (2016) divided the interaction behaviors into two dimensions: verbal and non-verbal behaviors. Verbal behaviors include restating one's own solutions, questioning and answering, doubting, commenting, evaluating, and so forth. Non-verbal behaviors include observation, taking notes, doing experiments, and any other action behaviors. We further classified the verbal behaviors into low-level cognitive (LC) and high-level cognitive (HC) behaviors (Table 1). Non-verbal behaviors like observing, taking notes, and doing experiments were not found in the current study. Instead, two new categories (i.e., solution submission and IT support) related to management behaviors were found in the current study. Therefore, the non-verbal behaviors were renamed as "management-related" behaviors. The coding scheme used for analyzing peer tutoring behaviors in collaborative learning of MPS is shown in Table 1.

Table 1 Coding Framework for Peer Tutoring Behaviors

Behaviors and Codes	Description
Low-level Cognitive (LC) Behaviors	
(C1) Explain or Elaborate	Peer tutors provide facts/insights, express opinions, elaborate solution procedures and explain the reasons behind
(C2) Question	Peer tutors ask questions
(C3) Answer or Comment	Peer tutors answer questions raised by group members or make comments on the solutions provided by tutees
High-level Cognitive (HC) Behaviors	
(C4) Doubt	Peer tutors doubt the tutee's points of view
(C5) Feedback	Peer tutors correct the tutee's misconceptions, make suggestions for improvement, and modify the comments or ideas presented by the tutees
(C6) Evaluate	Peer tutors organize tutees to evaluate learning content and effectiveness; praise/encourage the tutees
(C7) Extend	Based on the previous discussions, peer tutors initiate a new topic and/or extend it to other related areas
Management Behaviors	
(M1) Command	Peer tutors ask a tutee to follow the collaboration rules and/or to submit the worksheet, to present their solutions, etc
(M2) Guide or Assist	Peer tutors provide guidance/assistance when tutees encounter problems or difficulties
(M3) Demonstrate	Peer tutors use multimedia presentations, showcase individual/group work, or demonstrate to tutees how to present their work on the learning platform
(M4) Submit	Peer tutors submit individual/team members'/group learning work
(M5) IT support	Peer tutors provide technical support for tutees related to the use of the learning platform

The unit of analysis in this study was a learning event, which is a peer tutor's continuous behavior for a specific purpose. For example, during the consensus-reaching phase of the first period, T5 gave an opinion: "I think what you just said is wrong. This task mainly examines how to find out electricity bills for which different formulae will be used under different conditions, rather than simple decimal multiplication." This was coded as a feedback behavior. When organizing group members to share their solutions one by one, T2 said: "Your summary is great! Now, let's invite another classmate to present his solutions." Here T2 did two things, giving feedback to the peer who just made the presentation and giving a command to the next peer. Therefore, her behaviors were coded as feedback behavior and command behavior. A learning event can be a few words or a series of actions of the peer tutor; it may also be a single word. Nvivo11 (Dalkin et al., 2021) was used to analyze the video data. To ensure consistency in coding, each video was coded by two authors of this paper separately. The coding results were tested for consistency using SPSS 23.0, and the Kappa value was 0.81, indicating good consistency. For any inconsistencies, the two researchers watched the video together and reached an agreement through negotiation.

3.4 Data analysis

Quantitative methods including LSA and ANOVA were used in the current study for data analyses. LSA was used to estimate the probability of one behavior occurring after another for each of the peer tutors of the eight groups. This method has been widely used by educational researchers to identify sequences that occur more frequently in learning behavior data in order to identify learning behavior sequences that can promote learning effectiveness (Chiang, 2017; Wang et al., 2020). LSA was performed using Generalized Sequential Querier software GSEQ5.1 (https://www.mangold-international.com/en/products/software/gseq). The results obtained include behavioral frequency conversion tables, residual tables, and conversion plots with arrows to show the significant behavior sequences with a z-score larger than 1.96 (Bakeman & Quera, 2011) and Q not less than 0.30 (McComas et al., 2009). The use of LSA opens up the "black box" of the peer tutoring process and models, which allows us to further explore the effects of different tutoring styles on students' achievement. To answer the second research question, ANOVA was conducted to compare the performance of groups under the leadership of tutors with different tutoring models.

4 Results

The results include: (a) descriptive statistics of the tutors' behaviors; (b) three types of tutoring models based on conversion plots obtained from LSA; and (c) the performance differences in MPS among the groups with tutors using the three different tutoring models.

4.1 Descriptive statistics of the tutors' behaviors

Table 2 shows the frequency of each behavior category for all the peer tutors. In total, the eight peer tutors made 1,141 tutoring behaviors. Data in the last column of Table 2 revealed that the tutors engaged in management behaviors most frequently, followed by low-level cognitive behaviors, and, least frequently, high-level cognitive behaviors. This was true for each individual tutor as well, except for T1, who engaged in low-level and high-level cognitive behaviors equally frequently (38 versus 39 instances, respectively). More than half (53.81%) of the tutoring behaviors were related to management matters. In particular, the peer tutors tried to give commands in 35.14% of the learning events and to provide guidance and assistance in 15.07% of the learning events. Regarding the cognitive behaviors, there were more LC behaviors (27.96%) than HC behaviors (18.23%). Among the LC behaviors, asking questions (21.56%) accounted for the largest proportion, while answering questions (4.38%) and explaining behaviors (2.02%) were less frequent. Among the HC

Table 2 Frequency of Peer Tutoring Behaviors in Different Categories

Code	T1	T2	T3	T4	T5	T6	T7	T8	Total
Low-level Cognitive (LC) Behaviors									
C1	3	1	1	1	7	1	2	7	23 (2.02%)
C2	25	31	17	13	62	9	41	48	246 (21.56%)
C3	10	8	8	1	4	9	1	9	50 (4.38%)
Sub-total	38	40	26	15	73	19	44	64	319 (27.96%)
High-level Cognitive (HC) Behaviors									
C4	10	2	7	5	6	1	1	11	43 (3.77%)
C5	4	1	8	0	13	2	5	15	48 (4.21%)
C6	23	2	4	3	34	10	3	26	105 (9.20%)
C7	2	1	1	0	1	1	1	5	12 (1.05%)
Sub-total	39	6	20	8	54	14	10	57	208 (18.23%)
Management Behaviors									
M1	76	34	33	41	50	29	47	91	401 (35.14%)
M2	28	13	9	5	24	11	29	53	172 (15.07%)
M3	1	6	1	3	3	4	3	4	25 (2.19%)
M4	4	2	0	1	1	0	0	1	9 (0.79%)
M5	1	1	0	0	0	2	1	2	7 (0.61%)
Sub-total	110	56	43	50	78	46	80	151	614 (53.81%)
Total	187	102	89	73	205	79	134	272	1141 (100%)

C1 = Explain or Elaborate; C2 = Question; C3 = Answer or Comment; C4 = Doubt; C5 = Feedback; C6 = Evaluate; C7 = Extend; M1 = Command; M2 = Guide or Assist; M3 = Demonstrate; M4 = Submit; M5 = IT support

behaviors, evaluating behaviors (9.20%) was most frequently exhibited, and extending behaviors (1.05%) were least frequently exhibited.

The data in Table 2 revealed great differences in the number of behaviors among the eight peer tutors. Firstly, the total numbers of behaviors showed great differences, with T1, T5, and T8 as the top three and T3, T4, and T6 as the bottom three. T8's total number of behaviors was the highest and was 3.73 times T3's total. Secondly, the distributions in the three categories of behaviors across the eight tutors were not consistent. For example, management behaviors accounted for 68% of T4's total behaviors, and his HC behaviors accounted for only about 11% of the total. In contrast, T5's management behaviors accounted for 38% of her total, and her HC behaviors accounted for slightly more than one-fourth of her total. In terms of the percentages of the behaviors in the three categories, T4 was the highest and T5 was the lowest in management behaviors; T2 was the highest and T1 was the lowest in LC behaviors; T5 was the highest and T2 was the lowest in HC behaviors. Again, there were also great differences in each sub-category. For example, T4 tried to give commands in 56% of the learning events whereas T5 tried to give commands in only 24% of the learning events.

4.2 The three peer tutoring models

To describe the peer tutoring models of the eight groups, we first needed to draw the behavior sequence transition (BST) diagram for each tutor. Here, we use T1 as an example to show how that is done. Firstly, a table (Table 3) was made to show the BST frequencies where a cell shows the frequency that a column behavior occurred immediately after its corresponding row behavior and its corrected residual result. Finally, the BST diagram for T1 (Fig. 2) was drawn with each arrow to show the transition from an initial behavior to its sequential behavior together with its *z*-value.

One BST diagram was drawn for each tutor. Therefore, eight diagrams were obtained, and they can be further classified into the following three types from the least to the most complex:

(a) Type-I tutoring model, which only included T2 (Fig. 3), where transitions only existed between the management behaviors, and the LC and HC behaviors were all isolated. For these reasons, this type can be named as an "organization-oriented" peer tutor.
(b) Type-II tutoring model, which included T3, T4, T6, and T7 (Fig. 4), where transitions existed among LC and management behaviors, but the HC behaviors were isolated. Because the cognitive level of this type is supportive of the group members' learning, but it is not as high as that of Type-III, we labeled peer tutors who used this model "cognitive-supported" peer tutors.
(c) Type-III tutoring model, which included T1, T5, and T8 (Fig. 5), where the HC behaviors were not isolated and the transitions among LC, HC, and management behaviors were more complex. Therefore, we labeled peer tutors who used this model "cognitive-guided" peer tutors.

Organization-oriented peer tutor As shown in his BST diagram (Fig. 3), T2 asked the tutees to explain something after assisting them in submitting their work (M4 → M1), offered help to the tutees when they encountered difficulties (e.g., not knowing how to use the software) (M1 → M2), and provided assistance to the tutees continuously (M2 → M2).

Cognitive-supported peer tutors The BST diagram for T3 (Fig. 4) revealed that he actively responded to the tutees' questions while they were demonstrating their learning work (M3 → C3) and he explained or elaborated in more detail until the tutees understood (C3 → C1).

The BST diagram for T4 (Fig. 4) indicated that she offered help to the tutee(s) when they encountered difficulties after giving them instructions (M1 → M2). She

Table 3 Behavior Sequence Transition (BST) Frequencies (Corrected Residual Result) of T1

	C1	C2	C3	C4	C5	C6	C7	M1	M2	M3	M4	M5
C1	0 (-0.23)	0 (-0.69)	2 (**4.70****)	0 (-0.42)	0 (-0.26)	1 (1.09)	0 (-0.18)	0 (-1.46)	0 (-0.73)	0 (-0.13)	0 (-0.23)	0 (0.00)
C2	0 (-0.69)	6 (1.62)	0 (-1.29)	4 (**2.49****)	0 (-0.80)	3 (-0.09)	0 (-0.57)	10 (-0.11)	2 (-1.02)	0 (-0.40)	0 (-0.69)	0 (0.00)
C3	0 (-0.40)	1 (-0.23)	1 (0.76)	0 (-0.74)	0 (-0.46)	2 (0.90)	0 (-0.32)	5 (0.91)	0 (-1.28)	0 (-0.23)	0 (-0.40)	0 (0.00)
C4	0 (-0.42)	3 (1.55)	2 (**2.08****)	0 (-0.78)	1 (1.74)	1 (-0.25)	1 (**2.79****)	1 (-2.05)	1 (-0.44)	0 (-0.24)	0 (-0.42)	0 (0.00)
C5	0 (-0.26)	1 (0.67)	0 (-0.49)	0 (-0.49)	0 (-0.30)	0 (-0.77)	0 (-0.21)	3 (1.40)	0 (-0.84)	0 (-0.15)	0 (-0.26)	0 (0.00)
C6	0 (-0.66)	0 (-2.04)	3 (1.71)	0 (-1.23)	1 (0.76)	4 (0.75)	0 (-0.54)	12 (1.62)	2 (-0.88)	0 (-0.38)	0 (-0.66)	1 (0.00)
C7	1 (**5.42****)	0 (-0.57)	0 (-0.34)	0 (-0.34)	0 (-0.21)	0 (-0.54)	1 (**6.69****)	0 (-1.19)	0 (-0.59)	0 (-0.11)	0 (-0.18)	0 (0.00)
M1	1 (-0.25)	11 (0.39)	2 (-1.35)	4 (0.03)	1 (-0.64)	11 (0.77)	0 (-1.17)	25 (-1.63)	15 (1.73)	1 (1.22)	3 (**2.12***)	0 (0.00)
M2	1 (0.87)	3 (-0.49)	0 (-1.38)	1 (-0.48)	1 (0.54)	1 (-1.56)	0 (-0.60)	15 (1.47)	6 (1.08)	0 (-0.43)	0 (-0.74)	0 (0.00)
M3	0 (-0.13)	0 (-0.40)	0 (-0.24)	0 (-0.24)	0 (-0.15)	0 (-0.38)	0 (-0.11)	1 (1.20)	0 (-0.42)	0 (-0.07)	0 (-0.13)	0 (0.00)
M4	0 (-0.26)	0 (-0.80)	0 (-0.49)	1 (1.74)	0 (-0.30)	0 (-0.77)	0 (-0.21)	2 (0.37)	1 (0.58)	0 (-0.15)	0 (-0.26)	0 (0.00)
M5	0 (0.00)	0 (0.00)	0 (0.00)	0 (0.00)	0 (0.00)	0 (0.00)	0 (0.00)	0 (0.00)	0 (0.00)	0 (0.00)	0 (0.00)	0 (0.00)

Note. Corrected residual results (z-score) above 1.96 are in bold

$^{**}p < .01$.

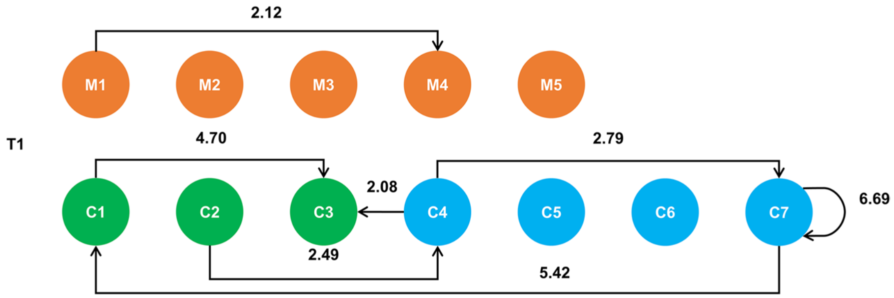


Fig. 2 The Behavior Sequence Transition (BST) Diagram of T1

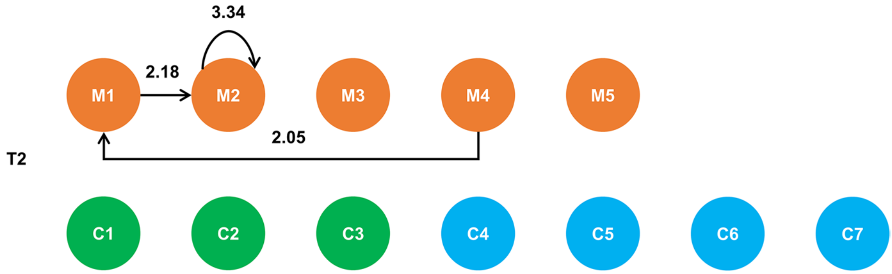


Fig. 3 The Behavior Sequence Transition (BST) Diagram of T2

demonstrated how to use the platform to present individual work after helping tutees to submit their work (M4 → M3) and then asked the tutees whether they had learned how to use the platform (M3 → C2). However, the tutees still did not know how to use it, so she performed the demonstration again (C2 → M3) until they fully mastered it.

The BST diagrams for T6 and T7 (Fig. 4) are very similar in that there were transitions between management behaviors and between LC behaviors. In particular, T6 preferred to explore the learning software independently and demonstrated to the tutees how to use the platform to assist in presenting their learning work (M5 → M3). Also, he answered the tutees' questions several times in succession (C3 → C3). T7 offered help to the tutees with difficulties after giving them instructions (M1 → M2), explained his solution to the group and answered questions from the group members (C1 → C3), and provided further explanations until the tutees understood (C3 → C1).

In summary, the peer tutors in these four groups provided support to their team members on how to use the platform when they had difficulties and gave their answers and/or explanations in response to the questions from the group members at low (but not high) cognitive levels.

Cognitive-guided peer tutors Since the leadership role of the peer tutors in this type is more complex than those in the previous two types, we shall follow the four-phase model of collaborative learning to organize the following discussion.

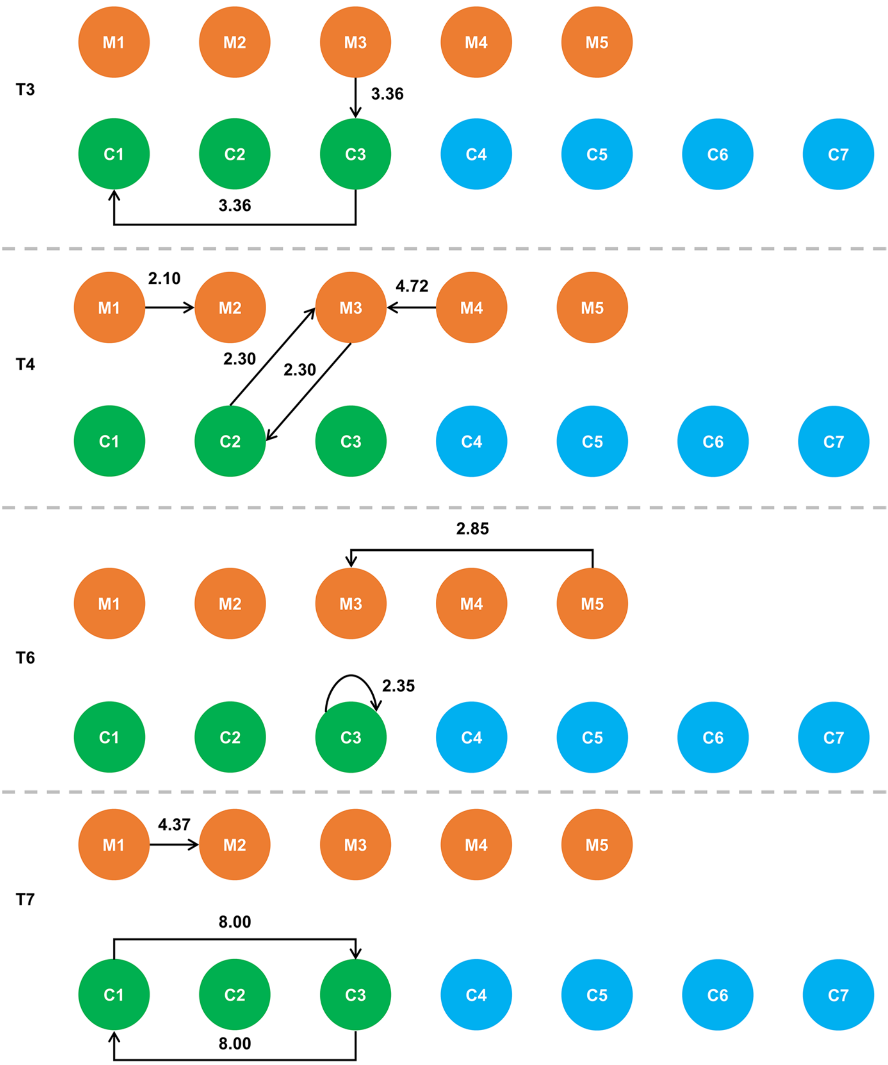


Fig. 4 The Behavior Sequence Transition (BST) Diagram of T3, T4, T6, and T7

The BST diagram for T1 (Fig. 5) indicated that in the preparation phase, T1 did the submission on behalf of the team members after issuing the command to submit their worksheets (M1→M4). In the information-sharing phase, he answered the questions from the group members regarding the solution he had just presented (C1→C3). In the conflict-negotiating phase, he initiated the discussion in depth with questions to the tutees (C2→C4) and took the responsibility to provide further explanations (C4→C3) or to talk with the group members individually to resolve the conflicts when there were controversial issues (C4→C7). In the consensus-reaching phase, he led the group to summarize the various solutions and the methods

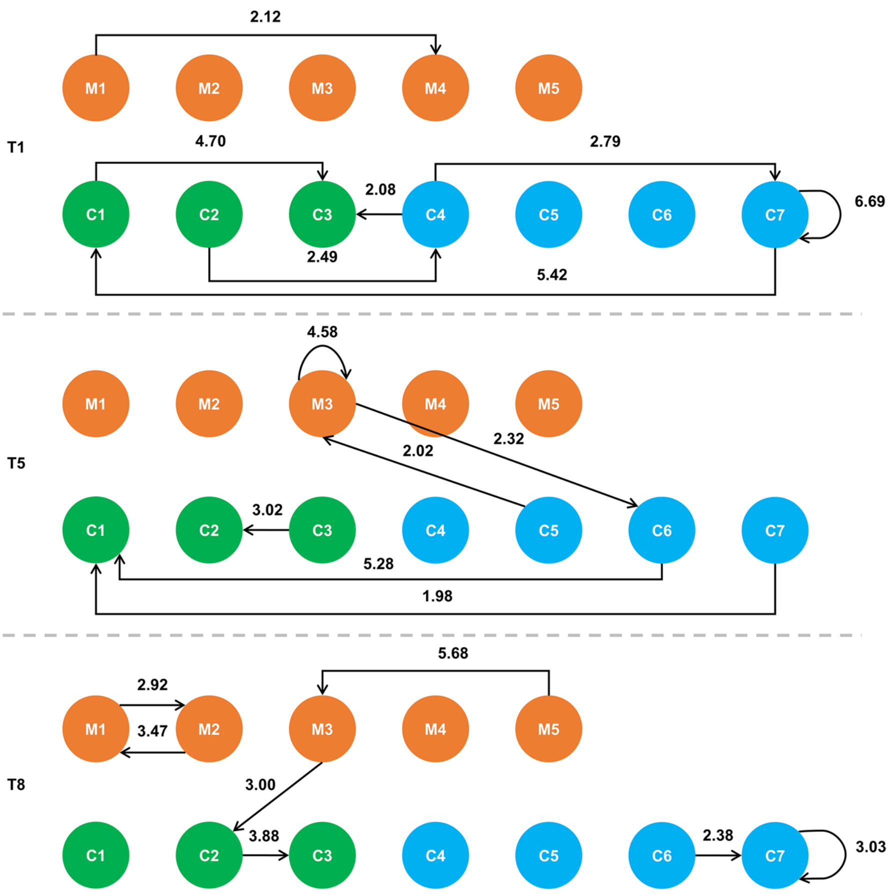


Fig. 5 The Behavior Sequence Transition (BST) Diagram of T1, T5, and T8

used for solving a problem. Meanwhile, he linked the methods with individual solutions (C7 → C1) or extended the new knowledge and/or methods to related topics (C7 → C7) in more depth and breadth.

The BST diagram for T5 (Fig. 5) revealed that she repeatedly demonstrated how to use the platform to display the worksheet in the preparation phase (M3 → M3). In the information-sharing phase, she answered questions from the team members and made links to the related mathematics knowledge applied in the problems through a series of questions (C3 → C2). In the conflict-negotiating phase, she helped the team members to correct their wrong answers, made suggestions for improvement, and presented the correct solution (C5 → M3). At the same time, she led the group to evaluate the problem-solving procedures (M3 → C6) and summarize the related knowledge (C6 → C1). In the consensus-reaching phase, she acted similarly to T1 (C7 → C1).

The BST diagram for T8 (Fig. 5) revealed that he provided IT support (e.g., he showed how to use the whiteboard, paintbrush, spotlight, and other tools in the platform) to group members through demonstrating how to use the platform to display the worksheet (M5→M3) in the preparation phase. Then he asked team members one by one to present their solutions and provided help to group members when needed (M1→M2 and M2→M1). In the information-sharing phase, he showed his solutions and organized group members to ask questions or questioned himself about his solution (M3→C2) and answered those questions (C2→C3). In the conflict-negotiating phase, he organized his group to summarize and evaluate the solutions and the related mathematics knowledge, and he resolved conflicts through discussion and debate (C6→C7). He also extended new knowledge and/or methods to related topics similarly to T1 (C7→C7).

In summary, the peer tutoring behavior sequences of T1, T5, and T8 were more complex than the other two types in the following ways: (a) Their peer tutoring behavior sequences covered the four phases of collaborative learning; (b) they all contained transition behaviors linking extending behaviors (C7) with low-level cognitive behaviors (C1) or from C7 to C7; (c) as group leaders, they often asked questions to the team members or themselves, and they provided answers and/or explanations to the questions (C2→C3 or C2→C1); (d) and they either did the management work themselves or showed the group members how to use the platform for presentation and discussion after issuing commands. The data in Table 2 also revealed that the total number of behaviors of Type-III peer tutors was the highest, and their number of behaviors in the HC category was also the highest. This means that the Type-III peer tutors did play a very important guiding role in the collaborative learning of MPS. The other two types of peer tutors were usually busy with maintaining group discipline or focusing on low-level cognitive activities.

4.3 Effect of different peer tutoring models on students' achievement

The second research question was to examine the impact of the three peer tutoring models on the groups' MPS performance. The mean scores and standard deviations

Table 4 Mean (Standard Deviation) of Students with Three Types of Peer Tutors

Peer Tutoring Models	Four Periods					Polya's MPS Steps		
	Period 1	Period 2	Period 3	Period 4	Mean	Step 1	Step 2	Step 3
Type-I: Organization-oriented	48.82 (29.49)	52.64 (27.94)	69.64 (26.61)	46.91 (22.00)	54.50 (21.01)	5.53 (1.56)	16.84 (6.87)	3.88 (2.44)
Type-II: Cognitive-supported	50.23 (23.82)	64.46 (22.87)	68.54 (18.72)	65.35 (24.82)	62.15 (17.74)	6.64 (1.45)	19.75 (6.70)	4.67 (1.86)
Type-III: Cognitive-guided	63.61 (22.39)	72.17 (16.34)	72.19 (20.23)	70.94 (20.62)	69.73 (14.22)	7.32 (1.15)	21.96 (4.83)	5.58 (1.95)
F value	3.635*	3.800*	0.336	4.600*	4.092*	2.996	3.316*	3.990*

*$p < .05$

of students under the three types of peer tutoring models in the four MPS periods and in the three steps of Polya's MPS are shown in Table 4.

Students' performance in the four MPS periods ANOVAs were conducted to compare the mathematics performance in each of the four periods. It was found that the performances among the students under the three types of tutoring models were significantly different at the 0.05 level except in Period 3. Post-hoc pair-wise comparisons were conducted for each period. In Period 1, the significant performance difference only existed between students under the Type-II and Type-III tutoring models (mean difference = -13.38, $p < 0.05$). In Period 2, the significant difference was between students under the Type-I and Type-III tutoring models (mean difference = -19.53, $p < 0.05$). In Period 4, there were significant differences at the 0.05 level between students under the Type-I and Type-III tutoring models (mean difference = -24.04) and between students under the Type-I and Type-II tutoring models (mean difference = -18.45).

Students' overall performance As shown in Table 4, the ANOVA analysis results showed that the mean scores of students under the three tutoring models were significantly different at the 0.05 level. Post-hoc pair-wise comparisons on the mean scores of students showed that there were significant differences between students under the Type-I and Type-III tutoring models (mean difference = -15.23, $p < 0.05$) and between students under the Type-II and Type-III tutoring models (mean difference = -7.58, $p < 0.05$). There was no significant difference between students under the Type-I and Type-II tutoring models ($p > 0.05$).

The data in Table 4 revealed that the mean score of students under Type-III peer tutors was higher and the SD was lower than those of students under tutors of the other two types. This means that the Type-III tutors not only helped their groups perform better but also helped them close the performance gaps between members.

Students' performance in the three steps of Polya's MPS Table 4 shows the means and standard deviations of students under the three tutoring models in the three steps of the modified Polya's MPS model. ANOVAs were conducted to compare their mathematics performance in each step. It was found that the performances among the students under the three types of tutoring models were significantly different in Steps 2 and 3 at the 0.05 level. Post-hoc pair-wise comparisons were also conducted to compare the performance of students under the three different types of tutoring models. In Step 1, a significant performance difference only existed between students under the Type-II and Type-III tutoring models (mean difference = -0.68, $p < 0.05$). In Step 2, only the difference between students under the Type-I and Type-III tutoring models was significant (mean difference = -5.12, $p < 0.05$). In Step 3, the differences between students under the Type-I and Type-III tutoring models (mean difference = -1.71) and between students under the Type-II and Type-III tutoring models (mean difference = -0.91) reached a significant level ($p < 0.05$). There was no significant difference between students under the Type-I and Type-II tutoring models. In summary, the students under the Type-III peer tutoring model performed better than the students under the Type-I peer tutoring model in Steps 2 and 3, and better than the students under the Type-II peer tutoring model in Steps 1 and 3.

5 Conclusion and Discussion

This study examined the in-class portion of flipped classroom instruction that was designed for group collaborative learning of MPS in a technology-enhanced learning environment. The study focused on the peer tutoring behavior patterns and the effect of those patterns on students' performance in MPS. We used the LSA method to visualize the peer tutoring behavior patterns. The main contribution of this study is the establishment of a coding framework that classifies the tutoring behaviors of peer tutors into LC, HC, and management behaviors, which made it possible to identify the three types of tutoring models based on the BST diagrams. Another contribution of the study is its examination of the effect of different peer tutoring models on students' performance in the different steps of Polya's MPS model rather than simply on their mean scores.

The results showed that the tutors' most frequent behaviors were management behaviors (53.81%). This finding is consistent with the findings of Berghmans et al. (2013) in that face-to-face tutoring in small groups might require more organizational and management behaviors. In terms of cognitive behaviors, peer tutors preferred to use questioning and evaluation strategies, but had much less HC behaviors (e.g., doubting, providing feedback, and making extensions). The cognitive level of peer tutors' guidance behaviors was generally low. Though we cannot expect fifth-grade tutors to consistently achieve high-level cognitive guidance, we can still do some work to enhance their leading role in collaborative learning activities, which will be further discussed in the next paragraphs.

This study identified three types of peer tutoring models: organization-oriented, cognitive-supported, and cognitive-guided. In the organization-oriented peer tutoring model (exhibited by T2), significant behavior sequences only existed between management behaviors (e.g., issuing commands, providing help, etc.). T2's LC and HC behaviors were isolated, although 40% of his behaviors were in the LC category (in particular, he frequently asked team members questions; see Table 2). He made a great effort to maintain classroom discipline and manage group activities. Compared with organization-oriented peer tutors, the cognitive-supported peer tutors' (T3, T4, T6, & T7) behavior sequences were not limited only to management behaviors, but included transitions between management and LC behaviors. However, their HC behaviors were still isolated from their management and LC behaviors. They did exhibit 8–20 HC behaviors with an average of 13, but this was much less than the number of HC behaviors exhibited by cognitive-guided peer tutors, whose average was 50. In the cognitive-guided peer tutoring models (T1, T5, & T8), the transitions between HC and management behaviors and between HC and LC behaviors were more complicated, especially in the case of T5. In addition, to make sure the collaborative learning activities moved forward smoothly, the cognitive-guided tutors also led the group discussion with both LC (e.g., explaining, questioning, and answering questions) and HC behaviors (e.g., doubting tutees' views, correcting their errors, evaluating and extending the learning topics, etc.). They made transitions among the three categories of behaviors frequently whenever it was necessary like real teachers.

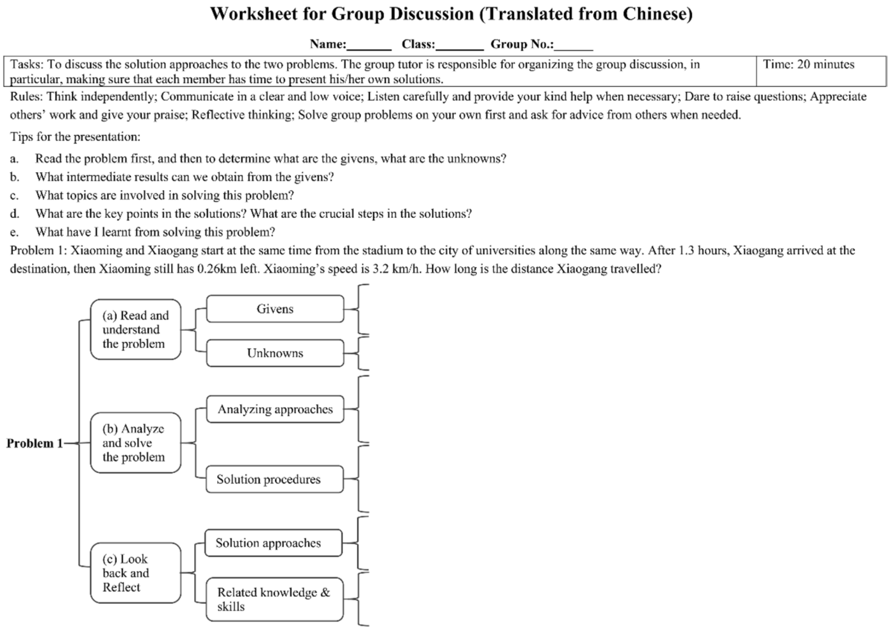
Cognitive-guided peer tutors didn't only exhibit more HC behaviors, but also made more complex transitions among LC, HC, and management behaviors, which was advantageous for their group members. Students under the cognitive-guided tutoring model performed significantly better than those under the organization-oriented and cognitive-supported tutoring models. The students under the cognitive-guided peer tutors performed better than those under the organization-oriented peer tutors in the modified Polya's second and third steps, and better than those under the cognitive-supported peer tutors in steps 1 and 3. The cognitive-guided tutors led their groups to improve their MPS skills through expanding the breadth and depth of collaborative discussions, which provided a good model for the peer tutors of the other two types to learn from.

The current study has some significant implications for classroom practice. Firstly, enough training needs to be provided to peer tutors. The peer tutors in the current study received 40-min of training in the first week, and only three of them showed the full range of behaviors and made transitions among the three categories of behaviors. Simply establishing a peer tutoring or peer-assisted learning environment will not guarantee a positive impact (Berghmans et al., 2014). Secondly, the training can be guided by Polya's four-step model so that the tutors can lead their groups to move smoothly from understanding the problem, through devising and implementing a solution plan, to reflecting on the solution methods and making extensions. As shown in the study, the cognitive-guided peer tutoring model had a significantly better impact on students' overall performance in MPS and the individual steps of the modified Polya's model. Another way to help peer tutors see their weaknesses and make improvements might be to watch videos of their own group and groups with cognitive-guided tutors and to compare the peer tutors' behaviors in the different phases of collaborative learning so that they know how to do better.

Although this study obtained some meaningful conclusions, there were still some limitations. Firstly, this study was conducted in the context of a flipped classroom with one peer tutor for each group of ten or eleven students. This ratio might cause a lot of challenges for the peer tutors. Further research could be conducted with smaller group sizes. Secondly, this study focused on the behaviors of peer tutors and the effect of their tutoring models on students' MPS performance. Exactly how the tutors' behaviors affect the group members needs further research that can identify the mechanism of this effect. Thirdly, the current study investigated the impact of tutoring models on the students' performance in different steps of Polya's MPS model. Future studies can be conducted to investigate the effect of tutoring behaviors at different Polya's steps on the performance of group members. Fourthly, the study intervention lasted only five weeks. Longitudinal studies could be designed to investigate the long-term effect of different tutoring models on students' mathematics achievement and their attitudes towards mathematics.

In summary, this research identified three types of peer tutoring models based on the BST diagrams drawn from LSA: organization-oriented, cognitive-supported, and cognitive-guided. ANOVA analysis revealed that the groups with cognitive-guided tutors who exhibited more HC behaviors performed significantly better than the other two groups. Furthermore, the students with cognitive-guided peer tutors performed better than the students with the organization-oriented peer tutors in Steps 2 and 3, and better than the students with the cognitive-supported peer tutors in Steps 1 and 3. This research provides a feasible path for the exploration and interpretation of peer tutoring models, which has practical implications for the organization of group learning activities in real classrooms.

Appendix. A worksheet for group discussion


Worksheet for Group Discussion (Translated from Chinese)

Name:______ Class:______ Group No.:______

Tasks: To discuss the solution approaches to the two problems. The group tutor is responsible for organizing the group discussion, in particular, making sure that each member has time to present his/her own solutions.	Time: 20 minutes

Rules: Think independently; Communicate in a clear and low voice; Listen carefully and provide your kind help when necessary; Dare to raise questions; Appreciate others' work and give your praise; Reflective thinking; Solve group problems on your own first and ask for advice from others when needed.

Tips for the presentation:

a. Read the problem first, and then to determine what are the givens, what are the unknowns?
b. What intermediate results can we obtain from the givens?
c. What topics are involved in solving this problem?
d. What are the key points in the solutions? What are the crucial steps in the solutions?
e. What have I learnt from solving this problem?

Problem 1: Xiaoming and Xiaogang start at the same time from the stadium to the city of universities along the same way. After 1.3 hours, Xiaogang arrived at the destination, then Xiaoming still has 0.26km left. Xiaoming's speed is 3.2 km/h. How long is the distance Xiaogang travelled?

Fig. 6 A worksheet for group discussion

Authors contribution All authors contributed to the study conception and design. Material preparation, data collection and analysis were performed by Yangyang Li, Zengzhao Chen, Jing Fang, and Chenyang Wang. The first draft of the manuscript was written by Yangyang Li. The review and editing of the manuscript was done by Xiuling He and Chunlian Jiang. All authors read and approved the final manuscript. This research was supported by National Natural Science Foundation of China (NSFC) for the Project "A Study on the Perception and Attribution Analysis of Learners' Higher-Order Thinking Activities" (No.: 61807012). We are grateful for the support from NSFC, National Intelligent Society Governance Experiment Base (Education), and Central China Normal University. Any opinions expressed herein are those of the authors and do not necessarily represent the funds' views.

Data availability The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.

Declarations

Conflict of Interests The authors declare that they have no conflict of interest.

References

Al-Zahrani, A. M. (2015). From passive to active: The impact of the flipped classroom through social learning platforms on higher education students' creative thinking. *British Journal of Educational Technology, 46*(6), 1133–1148. https://doi.org/10.1111/bjet.12353

Anderson, J. R. (1993). Problem Solving and Learning. *American Psychologist*, *48*(1), 35–44. https://doi.org/10.1037/0003-066X.48.1.35

Bakeman, R., & Quera, V. (2011). Sequential Analysis and Observational Methods for the Behavioral Sciences. *Cambridge University Press*. https://doi.org/10.1017/CBO9781139017343

Berghmans, I., Michiels, L., Salmon, S., Dochy, F., & Struyven, K. (2014). Directive versus facilitative peer tutoring? A view on students' appraisal, reported learning gains and experiences within two differently-tutored learning environments. *Learning Environments Research, 17*(3), 437–459. https://doi.org/10.1007/s10984-014-9168-8

Berghmans, I., Neckebroeck, F., Dochy, F., & Struyven, K. (2013). A typology of approaches to peer tutoring. Unraveling peer tutors' behavioural strategies. *European Journal of Psychology of Education, 28*(3), 703–723. https://doi.org/10.1007/s10212-012-0136-3

Bergmann, J., & Sams, A. (2012). *Flip your classroom: Reach every student in every class every day*. International society for technology in education.

Bhagat, K. K., Chang, C. N., & Chang, C. Y. (2016). The impact of the flipped classroom on mathematics concept learning in high school. *Journal of Educational Technology & Society, 19*(3), 134–142. https://www.jstor.org/stable/jeductechsoci.19.3.134.

Bishop, J. L., & Verleger, M. A. (2013). *The flipped classroom: A survey of the research*. Paper presented at 2013 ASEE Annual Conference & Exposition, Atlanta, Georgia.

Capstick, S., & Fleming, H. (2004). The learning environment of peer assisted learning. In: Peer Assisted Learning Conference, Bournemouth.

Chiang, T. H. C. (2017). Analysis of learning behavior in a flipped programing classroom adopting problem-solving strategies. *Interactive Learning Environments, 25*(2), 189–202. https://doi.org/10.1080/10494820.2016.1276084

Chu, H. C., Chen, J. M., & Tsai, C. L. (2017). Effects of an online formative peer-tutoring approach on students' learning behaviors, performance and cognitive load in mathematics. *Interactive Learning Environments, 25*(2), 203–219. https://doi.org/10.1080/10494820.2016.1276085

Colvin, J. W., & Ashman, M. (2010). Roles, risks, and benefits of peer mentoring relationships in higher education. *Mentoring & Tutoring: Partnership in Learning, 18*(2), 121–134. https://doi.org/10.1080/13611261003678879

Curriculum Planning and Development Division (CPDD). (2019). *Mathematics syllabuses secondary one to four express course, normal (academic) course*. Singapore: Ministry of Education. Retrieved May 20, 2022, from https://www.moe.gov.sg/-/media/files/secondary/syllabuses/maths/2020-express_na-maths_syllabuses.ashx?la=en&hash=E79043503E0EE64FA579D7514760663151459ED9

Curriculum Planning and Development Division (CPDD). (2020). *Mathematics syllabus primary one to six*. Singapore: Ministry of Education. Retrieved May 20, 2022, from https://www.moe.gov.sg/-/media/files/primary/mathematics_syllabus_primary_1_to_6.pdf

Dalkin, S., Forster, N., Hodgson, P., Lhussier, M., & Carr, S. M. (2021). Using computer assisted qualitative data analysis software (CAQDAS; NVivo) to assist in the complex process of realist theory generation, refinement and testing. *International Journal of Social Research Methodology, 24*(1), 123–134. https://doi.org/10.1080/13645579.2020.1803528

De Smet, M., Van Keer, H., & Valcke, M. (2008). Blending asynchronous discussion groups and peer tutoring in higher education: An exploratory study of online peer tutoring behaviour. *Computers in Education, 50*, 207–223. https://doi.org/10.1016/j.compedu.2006.05.001

Erbil, D. G. (2020). A review of flipped classroom and cooperative learning method within the context of Vygotsky theory. *Frontiers in Psychology, 11*, 1–9. https://doi.org/10.3389/fpsyg.2020.01157

Foldnes, N. (2016). The flipped classroom and cooperative learning: Evidence from a randomised experiment. *Active Learning in Higher Education, 17*(1), 39–49. https://doi.org/10.1177/1469787415616726

Galway, L. P., Corbett, K. K., Takaro, T. K., Tairyan, K., & Frank, E. (2014). A novel integration of online and flipped classroom instructional models in public health higher education. *BMC Medical Education, 14*(1), 1–9. https://doi.org/10.1186/1472-6920-14-181

Gong, D., Yang, H. H., & Cai, J. (2020). Exploring the key influencing factors on college students' computational thinking skills through flipped-classroom instruction. *International Journal of Educational Technology in Higher Education, 17*, 1–13. https://doi.org/10.1186/s41239-020-00196-0

González-Gómez, D., Jeong, J. S., & Rodríguez, D. A. (2016). Performance and perception in the flipped learning model: An initial approach to evaluate the effectiveness of a new teaching methodology in a general science classroom. *Journal of Science Education and Technology, 25*(3), 450–459. https://doi.org/10.1007/s10956-016-9605-9

Goodrich, A. (2018). Peer mentoring and peer tutoring among K–12 students: A literature review. *Update: Applications of Research in Music Education*, *36*(2), 13–21. https://doi.org/10.1177/8755123317708765

Hembree, R. (1992). Experiments and relational studies in problem solving: A meta-analysis. *Journal for Research in Mathematics Education, 23*(2), 242–273. https://doi.org/10.2307/749120

Hwang, G. J., & Chen, P. Y. (2019). Effects of a collective problem-solving promotion-based flipped classroom on students' learning performances and interactive patterns. *Interactive Learning Environments, 1–16*,. https://doi.org/10.1080/10494820.2019.1568263

Jdaitawi, M. (2020). Does Flipped Learning Promote Positive Emotions in Science Education? A Comparison between Traditional and Flipped Classroom Approaches. *Electronic Journal of e-Learning*, *18*(6), 516–524. https://doi.org/10.34190/JEL.18.6.004

Jiang, C., & Chua, B. L. (2010). Strategies for solving three fraction-related word problems on speed: A comparative study between Chinese and Singapore students. *International Journal of Science and Mathematics Education, 8*(1), 73–96. https://doi.org/10.1007/s10763-009-9163-1

Johnson, D. W., Johnson, R. T., & Stanne, M. B. (2000). Cooperative learning methods: A meta-analysis. Retrieved March 5, 2022, from https://www.researchgate.net/profile/David-Johnson-113/publication/220040324_Cooperative_learning_methods_A_meta-analysis/links/00b4952b39d258145c000000/Cooperative-learning-methods-A-meta-analysis.pdf

Lai, C. L., & Hwang, G. J. (2016). A self-regulated flipped classroom approach to improving students' learning performance in a mathematics course. *Computers & Education, 100*, 126–140. https://doi.org/10.1016/j.compedu.2016.05.006

Madaio, M., Cassell, J., & Ogan, A. (2017). "I think you just got mixed up": Confident peer tutors hedge to support partners' face needs. *International Journal of Computer-Supported Collaborative Learning, 12*(4), 401–421. https://doi.org/10.1007/s11412-017-9266-6

McComas, J. J., Moore, T., Dahl, N., Hartman, E., Hoch, J., & Symons, F. (2009). Calculating contingencies in natural environments: Issues in the application of sequential analysis. *Journal of Applied Behavior Analysis, 42*(2), 413–423. https://doi.org/10.1901/jaba.2009.42-413

McLaughlin, J. E., Griffin, L. M., Esserman, D. A., Davidson, C. A., Glatt, D. M., Roth, M. T., … & Mumper, R. J. (2013). Pharmacy student engagement, performance, and perception in a flipped satellite classroom. *American Journal of Pharmaceutical Education, 77*(9), 1–8. https://doi.org/10.5688/ajpe779196

Munir, M. T., Baroutian, S., Young, B. R., & Carter, S. (2018). Flipped classroom with cooperative learning as a cornerstone. *Education for Chemical Engineers, 23*, 25–33. https://doi.org/10.1016/j.ece.2018.05.001

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. Reston, VA, NCTM. Retrieved May 20, 2022, from https://www.rainierchristian.org/NCTM_principles-and-standards-for-school-mathematics.pdf

Nawaz, A., & Rehman, Z. U. (2017). Strategy of peer tutoring and students success in mathematics: An analysis. *Journal of Research and Reflections in Education, 11*(1), 15–30. https://ue.edu.pk/jrre/articles/1101002.pdf.

Nguyen, B., Yu, X., Japutra, A., & Chen, C. H. S. (2016). Reverse teaching: Exploring student perceptions of "flip teaching." *Active Learning in Higher Education, 17*(1), 51–61. https://doi.org/10.1177/1469787415616727

Organization for Economic Co-operation and Development (OECD). (2018). PISA 2021 Mathematics Framework (first draft). Retrieved May 15, 2022, from https://www.mokykla2030.lt/wp-content/uploads/2018/12/GB-2018-4-PISA-2021-Mathematics-Framework-First-Draft.pdf

Qiang, J. (2018). Effects of digital flipped classroom teaching method integrated cooperative learning model on learning motivation and outcome. *EURASIA Journal of Mathematics, Science and Technology Education*, *14*(6), 2213–2220. https://doi.org/10.29333/ejmste/102356

Sahin, A., Cavlazoglu, B., & Zeytuncu, Y. E. (2015). Flipping a college calculus course: A case study. *Journal of Educational Technology & Society, 18*(3), 142–152. https://www.jstor.org/stable/jeductechsoci.18.3.142.

Shi, Y., Ma, Y., MacLeod, J., & Yang, H. H. (2020). College students' cognitive learning outcomes in flipped classroom instruction: A meta-analysis of the empirical literature. *Journal of Computers in Education, 7*(1), 79–103. https://doi.org/10.1007/s40692-019-00142-8

Sun, J. C. Y., & Wu, Y. T. (2016). Analysis of learning achievement and teacher–student interactions in flipped and conventional classrooms. *International Review of Research in Open and Distributed Learning*, *17*(1), 79–99. https://doi.org/10.19173/irrodl.v17i1.2116

Sun, Z., Lin, C. H., Lv, K., & Song, J. (2021). Knowledge-construction behaviors in a mobile learning environment: A lag-sequential analysis of group differences. *Educational Technology Research and Development, 69*(2), 533–551. https://doi.org/10.1007/s11423-021-09938-x

Tong, H. (2017). 混合学习环境中协作知识建构策略研究 [Research on cooperative knowledge construction strategy in mixed learning environment](pp. 65–66). *Publishing house of electronics industry*.

Tucker, B. (2012). *The flipped classroom. Education Next, 12*(1), 82–83. https://lists.tki.org.nz/sympa/arc/ictenglish/2014-02/msg00032/ednext_20121_BTucker_flippedclassroom.pdf.

Wang, C., Fang, T., & Gu, Y. (2020). Learning performance and behavioral patterns of online collaborative learning: Impact of cognitive load and affordances of different multimedia. *Computers & Education, 143*, 1–14. https://doi.org/10.1016/j.compedu.2019.103683

Wang, K., & Zhu, C. (2019). MOOC-based flipped learning in higher education: Students' participation, experience and learning performance. *International Journal of Educational Technology in Higher Education, 16*(1), 1–18. https://doi.org/10.1186/s41239-019-0163-0

Wang, Y. L., Cheng, Y., Wang, F., Tong, S. H., & Huang, K. B. (2016). 技术支持下的课堂教学行为观察方法探究 [Research on the observation method of classroom teaching behavior supported by technology]. *Modern Educational Technology, 26*(09), 39–45.

Wei, X., Cheng, I., Chen, N. S., Yang, X., Liu, Y., Dong, Y., & Zhai, X. (2020). Effect of the flipped classroom on the mathematics performance of middle school students. *Educational Technology Research and Development, 68*(3), 1461–1484. https://doi.org/10.1007/s11423-020-09752-x

Zheng, L., Bhagat, K. K., Zhen, Y., & Zhang, X. (2020). The Effectiveness of the Flipped Classroom on Students' Learning Achievement and Learning Motivation: A Meta-Analysis. *Journal of Educational Technology & Society, 23*(1), 1–15. https://www.jstor.org/stable/26915403.

Zulaiha, Z., Bharata, H., & Dahlan, S. (2020). Developing Discovery Learning Assisted Peer Tutors to Improve Problem Solving. *International Journal of Trends in Mathematics Education Research, 3*(2), 106–111. https://ijtmer.saintispub.com/index.php/ijtmer/article/view/79.

Authors and Affiliations

Yangyang Li[1,2] · Chunlian Jiang[3] · Zengzhao Chen[1,2] · Jing Fang[1,2] · Chenyang Wang[1,2] · Xiuling He[1,2]

Yangyang Li
nercellyy@ccnu.edu.cn

Chunlian Jiang
cljiang@um.edu.mo

Zengzhao Chen
zzchen@ccnu.edu.cn

Jing Fang
fangjing@ccnu.edu.cn

Chenyang Wang
wangchenyang@mails.ccnu.edu.cn

1 Faculty of Artificial Intelligence in Education, Central China Normal University, Wuhan, China

2 National Engineering Research Center for Educational Big Data, Central China Normal University, Wuhan, China

3 Faculty of Education, University of Macau, Macao, China