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Coskewness and reversal of momentum returns: The US and international evidence

Liang Dong, Yiqing Dai, Tariq Haque, Hung Wan Kot, Takeshi Yamada

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Coskewness and Reversal of Momentum Returns:

The US and International Evidence

Liang Dong
School of Finance
Hunan University of Technology and Business
569 Yue Lu Ave., Chang Sha, China
liang.dong@hutb.edu.cn

Yiqing Dai
Independent
kevin.yiqingdai@gmail.com

Tariq Haque
Adelaide Business School
The University of Adelaide
10 Pulteney St., Adelaide SA 5005, Australia
tariq.haque@adelaide.edu.au

Hung Wan Kot
Faculty of Business Administration
University of Macau
Avenida da Universidade, Taipa, Macau
hungwankot@um.edu.mo

Takeshi Yamada
Research School of Finance, Actuarial Studies, and Statistics
Australian National University
26C Kingsley St., Acton ACT 2600, Australia
takeshi.yamada@anu.edu.au

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Abstract

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The winner-minus-loser (WML) momentum strategy carries an inherent downside as its returns have negative coskewness. We propose a coskewness-volatility-managed momentum strategy that reduces a large reversal risk of the baseline WML strategy and the volatility-managed momentum strategy by Barosso and Santa-Clara (2015) by up to 58% and 11%, respectively. The returns of our strategy generate a slightly positive skewness in contrast with the negative skewness of the WML and volatility-managed strategies. Since the coskewness of momentum portfolio returns predict future returns for up to 12 months, our strategy is effective for momentum portfolios with holding periods longer than one month. Our strategy also mitigates momentum downside risks in major international stock markets such as the UK, Germany, and France.

Keywords: Reversal risk; Coskewness; Momentum

JEL Codes: G12 G15

Highlights (for review)

Highlights

- Coskewness of momentum portfolio returns predicts stock returns up to 12 months.
- Coskewness of winner-minus-loser (WML) momentum returns indicates downside risk.
- Coskewness of WML portfolio returns predict momentum reversals after a bear market.
- We augment the strategy of Barroso and Santa-Clara (2015) with coskewness.
- The augmented momentum strategy mitigate downside for most international markets.

Coskewness and Reversal of Momentum Returns: The US and International Evidence

Abstract

The winner-minus-loser (WML) momentum strategy carries an inherent downside as its returns have negative coskewness. We propose a coskewness-volatility-managed momentum strategy that reduces the reversal risk of the baseline WML strategy by 61% and that of the volatility-managed momentum strategy (Barosso and Santa-Clara, 2015) by 20% for US stocks. The returns of our strategy generate a slightly positive skewness in contrast with the negative skewness of the WML and volatility-managed strategies. Since the coskewness of momentum portfolio returns predict future returns for up to 12 months, our strategy is effective for momentum portfolios of holding periods longer than one month. Our strategy also mitigates momentum downside risks in major international stock markets such as the UK, Germany, and France.

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1. Introduction

As one of the best-known anomalies in financial markets, the momentum strategy exploits the tendency of firms with high returns over the past three to twelve months to continue to outperform firms with low returns over similar periods (Jegadeesh and Titman, 1993). After almost 30 years, the momentum strategy remains a robust anomaly not captured by well-known factor models (Fama and French, 2015; Hou et al., 2020). Numerous studies have shown that the momentum anomaly also exists in other asset classes (Moskowitz et al., 2012; Menkhoff et al., 2012) and equity markets worldwide (Griffin et al., 2005; Asness et al., 2013). Some studies have examined the drivers of momentum profits (Novy-Marx, 2015; Ehsani and Linnainmaa, 2019; Kelly et al., 2021; Luo et al., 2021) and others have examined how to enhance the profitability of momentum strategies by adding more conditions (Hong et al., 2000; Avramov et al., 2007; Yang and Zhang, 2019).

However, despite its remarkable profitability, the momentum strategy has a left-skewed distribution, or large downside risk, compared with that of market returns and many other strategies. An earlier study by Harvey and Siddique (2000) showed that past winners have negative skewness, whereas past losers have less negative or even positive skewness, indicating the negative skewness of the Winners-Minus-Losers (WML) strategy and its potential downside risk. Recent studies (Barroso and Santa-Clara, 2015; Daniel and Moskowitz, 2016) highlight that a baseline WML strategy, though generating high average monthly returns, can give rise to momentum crashes that are infrequent but substantial: the long-short momentum strategy experienced a crash of -91.59% over two months in 1932 and a crash of -73.42% over three months in 2009.

The success of the momentum strategy relies on the continuation of past returns, whereas the magnitude of its return reversal demonstrates its downside risk. In this paper, we propose a momentum trading strategy that adjusts its downside risk using the momentum portfolio's exposure to negative coskewness. A coskewness is the covariance of an asset return and squared market returns (i.e., market volatility) when returns are expressed as standardized returns. A negative coskewness implies that market volatility is more closely associated with negative returns than positive ones. Harvey and Siddique (2000) show that the coskewness of winner portfolio returns is more negative than that of loser portfolio returns, which implies that the WML strategy has an inherent downside risk. Our strategy also takes advantage of the return predictability of the coskewness of momentum portfolio returns.

Beginning with the Fama-MacBeth (1973) (FM) regression of individual stocks, we show that the return coskewness of a momentum portfolio significantly predicts the monthly cross-sectional returns of individual stocks included in the portfolio; the more negative the coskewness is, the higher the expected returns. Also, we find that the return coskewness of momentum portfolios predicts individual stock returns significantly better than the prior 11-month returns of individual stocks for longer periods. To shed light on the driver of our findings, we stratify the cross-section of individual stocks into two groups; those that have extreme prior 11-month returns (in the top 10% and the bottom 10%) and the rest. We find that our results are mainly driven by stocks that have extreme prior returns.

Moreover, when we compare the return predictability of the coskewness of prior momentum portfolio returns with that of prior individual stock returns, we find that the latter only significantly predicts future returns for only one month, whereas the former significantly predicts future returns up to 12 months forward. In the mean-variance-skewness framework of asset pricing, rational investors prefer assets whose return distribution is less negatively-skewed. Thus, the seminal papers by Kraus and Litzenberger (1976, 1983) and Harvey and Siddique (2000) show that coskewness is a risk factor where investors require a risk premium for negatively-coskewed stocks. However, it is arguable if the coskewness of momentum portfolio returns reflects a rationally priced risk. Several studies show that momentum profits cannot be justified with a risk-based explanation (Fama and French, 1996; Moskowitz, 1999; Cooper et al., 2004). Since we find that the coskewness of momentum portfolio returns mainly affects stocks that have extreme past returns, our evidence suggests that investor biases might cause such extreme return reversals. For example, because winner stocks might overreact to negative news more strongly than loser stocks, the coskewness of winner stocks is more negative than that of loser stocks, resulting in greater reversals compared to losers.[1]

On the other hand, given the evidence of reversals in momentum portfolio returns, Daniel and Moskowitz (2016) point to the option-like feature of momentum payoffs; investing in a momentum strategy is like writing a call option and receiving an option premium. The statistically significant coefficient of coskewness in our FM regression indicates higher expected returns to constituent stocks in a momentum portfolio that has more negative coskewness. Thus, our finding also implies that the success of the momentum anomaly could be partially attributable to the option premium for extreme return reversals.

Following the line of momentum return reversals, Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016) find that momentum crashes tend to occur when the market rebounds following market declines and when market volatility is high. Such events, leading to negative skewness, make the momentum strategy less appealing to investors. To avoid momentum crashes, studies such as Barosso and Santa-Clara (2015), Daniel and Moskowitz (2016), and Daniel et al. (2019) develop various momentum strategies to manage downside risk, resulting in

[1] Also, such a pattern is in line with the disposition effect that investors tend to sell assets with increased value while keeping assets that have fallen.

superior investment performance. We present a simple trading strategy that augments that of Barosso and Santa-Clara (2015) (hereafter, BSC) by reducing the coskewness exposure. The BSC strategy manages the exposure to downside risk using the predicted volatility of WML returns. Their strategy reduces the exposure when the WML strategy's return volatility is predicted to be greater, improving the Sharpe ratio of the baseline WML strategy by around 80%. Our augmented strategy further mitigates the downside risk of the momentum strategy by adjusting the exposure of the WML portfolio to its coskewness, like the way the BSC adjusts the exposure to the baseline momentum strategy. Based on the time-series dynamics of the coskewness, our strategy reduces the momentum exposure when the coskewness becomes less negative and vice versa. While adjusting the exposure to downside risk, our strategy exploits the fact that a more negative coskewness predicts higher future returns. Thus, our strategy provides an additional downside hedge to the BSC strategy. Although our strategy might not greatly improve the Sharpe ratio of the BSC strategy, it considerably mitigates downside risk as the maximum drawdown is reduced by up to 61% and 20% from the baseline WML and BSC strategies, respectively, for the one-month holding period.

Another interesting finding relates to the length of the holding period for the momentum strategy. According to our FM regressions, the coskewness of momentum portfolio returns significantly predicts the cross-section of individual stock returns for up to 9 months forward. The result suggests that more negative coskewness predicts higher returns for up to 9 months despite the downside risk. Based on this finding, we increase the holding period of the momentum strategy by up to 12 months, which reduces the frequency of portfolio rebalancing. With our trading strategy, we find that the largest improvements of maximum drawdown occur for the 3- and 6-

month holding periods, reflecting the fact that coskewness predicts future returns most significantly for these periods.

Finally, we examine the robustness of the results using data from major international markets, i.e., the United Kingdom, France, Germany, and Japan. Our FM regressions show that coskewness of momentum portfolio returns predict more distant future returns better than prior 11-month returns, for the four international markets. Moreover, we find that our strategy is effective in all markets except for the Japanese market, where previous research has shown that the momentum effect does not exist or has been weak.

Our paper contributes to the branch of literature on enhancing the performance of the momentum strategy with dynamic risk management such as Barroso and Santa-Clara (2015), Daniel and Moskowitz (2016), and Daniel et al. (2019). Our paper also contributes to the coskewness literature in asset pricing. Kraus and Litzenberger (1976) and Harvey and Siddique (2000) studied how preference for positive skewness can generate a risk premium on negatively skewed assets. Chang et al. (2013) find that the cross-section of stock returns has substantial exposure to the risk captured by higher moments of market returns. Jondeau et al. (2019) find that the average skewness across firms well predicts future market returns. Among the literature, Harvey and Siddique (2000) claimed that skewness preference might explain the momentum effect. Our paper relates the momentum crash risk to the coskewness of winner and loser portfolio returns which are driven by stocks that have extreme past returns.

The paper proceeds as follows. Section 2 presents a summary of momentum portfolio returns and coskewness. In Section 3, we use the FM regressions to estimate the impact of the coskewness of momentum portfolio returns on individual stock returns. Section 4 discusses the relationship between coskewness and WML portfolio returns. Section 5 lays out the coskewness augmented momentum strategy. In Section 6, we examine if our findings also hold for international stocks. Section 7 concludes.

2. Momentum portfolio returns and coskewness

We use the CRSP dataset of individual US stocks, covering the period from January 1963 to December 2019. The sample contains all stocks listed on NYSE, AMEX, and NASDAQ, which have the available market and financial data for constructing the key variables used in this study. We compute the coskewness of the momentum portfolio (CS_PORT) by using all available monthly historical returns starting from January 1964. As coskewness is largely driven by tail events, we use all available return data (at least 120 months, starting from January 1964) with an expanding window up to the current month to capture rare events.[2] Following Harvey and Siddique (2000), we construct a direct measure of coskewness as follows:

$$CS_{p,t} = \frac{1}{t}\sum_{\tau=1}^{t}\frac{(r_{p,\tau}-\hat{\mu}_{p,t})(r_{m,\tau}-\hat{\mu}_{m,t})^2}{\hat{\sigma}_{p,t}\hat{\sigma}^2{}_{m,t}} \tag{1}$$

where $CS_{p,t}$ is coskewness for portfolio p computed using observations up to month t. Portfolio p corresponds to each momentum portfolio; $r_{p,\tau}$ is the excess return from the risk-free rate of portfolio p in month τ and $\hat{\mu}_{p,t}$ is the mean computed using observations up to month t; $r_{m,\tau}$ is the month τ excess market return from the risk-free rate and $\hat{\mu}_{m,t}$ is the mean computed using observations up to month t; $\hat{\sigma}_{p,t} = \sqrt{\sum_{\tau=1}^{t}(r_{p,\tau}-\hat{\mu}_{p,t})^2/t}$ and $\hat{\sigma}^2_{m,t} = \sum_{\tau=1}^{t}(r_{m,\tau}-\hat{\mu}_{m,t})^2/t$. Equation (1) shows that coskewness measures the association of portfolio return and market volatility, where the measure

[2] Because an accurate estimation of the coskewness requires sufficient historical data to capture extreme events of past crashes, we employ the expanding estimation window. We do not use a short rolling window with daily returns because it would not capture the major momentum crashes in the past. Similarly, the minimum requirement of 120 months' observations serves the same purpose. Though such a requirement would exclude some young firms from the FM regression, we find that the regression results remain similar when we shorten the minimum requirement to 36 months. In addition, note that young firms are included in the trading strategy analyses in Sections 4-6, where the minimum observation requirement is applied to WML returns, not to individual stock returns.

is unit-free and is like a correlation coefficient.[3] To control for the size effect and obtain a wider dispersion of CS_PORT values, we form 100 size-momentum double sorted portfolios each month, calculate the post-ranking value-weighted portfolio monthly returns, and estimate the portfolio coskewness using equation (1). After that, we assign the estimated portfolio coskewness to the constituent stocks within each portfolio, which is a common method for stock beta estimation (e.g., Fama and French, 1992).

To examine the characteristics of stocks with weak to strong momentum levels, we construct decile momentum portfolios by sorting stocks according to the prior 11-months returns and rebalancing monthly. Table 1 presents the summary statistics of the 10 momentum portfolios formed by ranking stocks using 11-month cumulative returns from the prior 2 to 11 months (PR). The CS_PORT reported in Table 1 is the average coskewness of the 10 momentum portfolios in each momentum decile. In Table 1, Panel A, we find that the coskewness of momentum portfolio returns is negatively related to PR; stocks that have higher prior returns tend to have more negative coskewness. Particularly, CS_PORT captures the reversals of portfolio returns whose constituent stocks are selected according to their past returns. Therefore, it is different from the reversals of individual stock returns. For comparison, we show the average coskewness of individual stock returns (CS_STK) for each momentum group. We calculate CS_STK using the historical monthly returns of individual stocks using equation (1). We not only find that the average CS_STK for each group has similar values but is also less negative (smaller magnitude) than most groups' average

[3] If $\hat{\sigma}^2_{m,t}$ is replaced with $\hat{\sigma}_{r_m^2,t}$, Equation (1) becomes the correlation coefficient between return and market volatility. Also, coskewness can be written as a volatility-weighted standardized excess return; $CS_{p,t} = \sum_{\tau=1}^{t} w_{\tau,t}\{(r_{p,\tau} - \hat{\mu}_{p,t}) / \hat{\sigma}_{p,t}\}$ where $w_{\tau.t} = (r_{m,\tau} - \hat{\mu}_{m,t})^2 / \sum_{\tau=1}^{t}(r_{m,\tau} - \hat{\mu}_{m,t})^2$ and $\sum_{\tau=1}^{t} w_{\tau,t} = 1$.

CS_PORT.[4] The CS_PORT measure better reflects the momentum's coskewness and fits the primary goal of our paper which examines the relationship between momentum return and its coskewness.[5]

As the correlations of individual stock returns in each group determine the standard deviation of momentum portfolio returns (STD), a higher standard deviation would make CS_PORT less negative, and vice versa. For example, the highest STD of the P1 (Loser) portfolio, 8.33%, suggests that the stocks in the loser portfolio not only have high standard deviations but also might have more positive correlations with each other. Despite having the most negative CS_PORT, the P10 (Winner) portfolio has a relatively large STD of 6.16%. Both P1 and P10 portfolios have larger STDs than the P2 to P9 portfolios, suggesting that extreme portfolios include stocks that might have a large systematic component relative to the idiosyncratic component. Also, we find that the loser portfolio returns are positively skewed (skewness=1.18), whereas the winner portfolio returns are negatively skewed (skewness=-0.57), suggesting that the WML strategy would have a negatively skewed return distribution. Finally, in Table 1, Panel A, we find that stocks that have relatively higher PR tend to have a higher book-to-market ratio (BM), lower maximum daily returns (MAX), and lower illiquidity (ILLIQ) than stocks that have lower PR.

<Insert Table 1>

Table 1, Panel B, shows the post-ranking returns for each momentum portfolio in the future 1st, 3rd, 6th, 9th, and 12th months. The return difference between winners and losers (P10-P1) is the highest in the first month (F=1), gradually declines, and becomes negative in the twelfth month

[4] Let N be the number of stocks in an equal weighted portfolio p where j=1…N, then we can write CS_PORT = $\mathrm{cov}(r_p/\sigma_p, (r_m/\sigma_m)^2) = \sum\mathrm{cov}(r_j/\sigma_p, (r_m/\sigma_m)^2)/N$ and CS_STK = $\sum\mathrm{cov}(r_j/\sigma_j, (r_m/\sigma_m)^2)/N$ where the summation is over N. If $\sigma_p^2 < (1/N)\sum\sigma_j^2$, then |CS_PORT| > |CS_STK| is likely. For simplicity, the returns are expressed in excess of the mean.
[5] Though not estimated with individual stocks' past returns, the CS_PORT measure is not entirely unrelated with each stock's past returns. Most likely, it is associated with the stock's past performance during months leading up to a portfolio-forming month, as a stock in the winner or loser portfolio has a high probability to remain as one of the winners or losers in the portfolio-forming month.

(F=12), indicating that longer periods might expose the WML strategy to lower returns and the risk of reversals.

To better understand and visualize the cross-sectional correlation between PR, CS_STK, and CS_PORT, we present a 3D surface plot of the three variables for the 100 portfolios sorted by size and past returns. From Figure 1, we find that PR and CS_PORT have an inverse relationship on average. That is, stocks with high past returns tend to have lower (more negative) CS_PORT. The two coskewness measures, CS_STK and CS_PORT, are slightly positively related. Lastly, we do not observe an obvious pattern between PR and CS_STK.

<Insert Figure 1>

3. Cross-section of stock returns and coskewness of momentum portfolio returns

This section presents FM regressions of individual stock returns where we are interested in the coskewness of momentum portfolio returns, CS_PORT, and prior 11-month returns, PR. Since CS_PORT is calculated for each post-ranking momentum portfolio, all individual stock returns in the corresponding portfolio are regressed on the portfolio coskewness. We also include the coskewness of individual stock returns, CS_STK, in the regression. The regressions also include the natural log of stock market value at the end of the previous month (MV); the beta coefficient computed from the market model (BETA);[6] a book-to-market ratio calculated using the previous month's market value and the previous financial year's book equity (BM); operating profit in the previous financial year (OP); asset growth rate from the previous two years (AG); the maximum daily return in the previous month (MAX) and the average daily Amihud (2002) ratio in the previous three months (ILLIQ). All explanatory variables are standardized to have zero mean and

[6] Beta is estimated using all observations available up to the month t in the market model.

unit standard deviation in the FM regressions in this paper to make the regression coefficients of the variables comparable. To be conservative, we use the Newey West standard errors to calculate the *t*-stats. The lags are selected as the count of forwarding months. For example, for the forward third month's return (F=3), the lag is 3. Untabulated results show that the normal *t*-stats are close to the Newey West *t*-stats, being slightly higher. Moreover, we also examined the robustness of the regression results by removing the 1% and 2% smallest stocks from the sample and using coarser 5 × 5 and 7 × 7 sorts to form the size-PR double sorted portfolios. The results are all similar to the main results reported in the paper.

<Insert Table 2>

Table 2 reports the results of FM regressions for monthly returns of individual stocks for the 1st, 3rd, 6th, 9th, and 12th post-ranking months. First, we find that the coskewness of individual stock returns (CS_STK) significantly predicts 1-month post-ranking returns with a negative coefficient (at the 5% significance level). The result is consistent with the earlier studies of Kraus and Litzenberger (1983) and Harvey and Siddique (2000), which suggest that coskewness is a risk factor that reflects investor preference for right-skewness. Our main variable, the coskewness of momentum portfolio returns, CS_PORT, significantly predicts future returns up to 9 months, at least at the 5% significance level, but not for 12 months. The prior 11-month cumulative returns, PR, only predict 1-month future returns at the 1% significance level and 3-month future returns at the 10% significance level.

For 1-month post-ranking returns (F=1), the coefficient of PR, 0.2148, is significant at the 1% significance level without the CS_PORT. However, the coefficient becomes 0.1609 when CS_PORT is added to the regression, suggesting that both return continuation and the momentum reversal explain 1-month future return. In addition, for 3-, 6-, and 9-month forward returns, we find that the coefficient of coskewness remains statistically significant, whereas that of PR becomes insignificant. Thus, our result shows that CS_PORT not only has more predictive power for future returns but also at longer horizons than PR.

Among other variables, we find that the following variables predict future returns up to 9 to 12 months; they are operating profit (OP), asset growth (AG), and maximum daily return in the previous month (MAX). On the other hand, market size (MV) predicts just 1-month forward returns, and illiquidity (ILLIQ) and book-to-market (BM) only predict up to 3 and 6 months, respectively.

To further examine what drives our main results, we stratify the sample into those with extreme prior 11-month returns, PR falling in the top 10% and the bottom 10% (i.e., tail stocks), and the rest (i.e., non-tail stocks). Table 3 shows FM regressions for the stratified samples. We find that the coefficients of CS_PORT for the tail stocks in Panel A are not only larger but also more significant (from F=1 to F=9) than those in Panel B for non-tail stocks. This suggests that the pricing effect of momentum coskewness is prominent for stocks with extreme past returns, i.e., the winner and loser stocks. On the other hand, we find opposite results for the coefficients of PR in Panel B for non-tail stocks; they are not only larger but also more significant (from F=1 to F=6) than those in Panel A, showing the pricing effect of past return is stronger for non-tail stocks than for tail stocks. Thus, our result not only shows that tail stocks drive our main results but also suggests that the momentum strategy that relies on return continuation is supported by the return predictability of momentum coskewness for winners and losers.

<Insert Table 3>

Next, we examine if the predictive power of CS_PORT is due to returns in volatile markets. Table 4, Panel A, shows results from a sample that consists of months that fall in the top 20% of months by market volatility (i.e., volatile periods), whereas Panel B shows results from the rest of the sample (i.e., less volatile periods). In Panel A, we find that the coefficient of PR is negative and statistically significant up to 12 months forward, indicating future return reversals during volatile periods. The coefficient of CS_PORT is insignificant, suggesting that the pricing effects of momentum coskewness might not exist during volatile periods.

In Panel B, we find that a significant coefficient of coskewness (CS_PORT) is only found during less volatile periods, which is statistically significant up to 12 months ahead. In comparison, the past return (PR)'s coefficient is only significant up to 3 months forward. This indicates that the pricing effect of coskewness on stock returns is more pronounced when the market is less volatile. Moreover, since large momentum return reversals tend to occur around volatile periods, our result suggests that coskewness could indicate such reversals in less volatile periods ahead of when they might happen.

<Insert Table 4>

4. Momentum strategy and coskewness

In this section, we examine the dynamic relation between WML and the coskewness of momentum portfolio returns. Figure 2 shows the cumulative return of the WML strategy, the cumulative return of excess market return, and market volatility during 1974-2019. The figure shows that momentum crashes tend to occur when market volatility is high. Indicated by the shaded area in the figure, we observe large WML return reversals for 2000.12-2001.1 (-51%), 2002.9-2003.6 (-47%), 2009.2-2009.9 (-76%), 2016.1- 2016.4 (-24%), and 2018.12-2019.2 (-14%). Barosso and Santa-Clara (2015) and Daniel and Moskowitz (2016) show that the WML strategy generates a high Sharpe ratio despite such crashes. Previous studies have documented that the strategy has a negative average beta risk, and its return has negative skewness and high kurtosis, suggesting infrequent and large reversals.

<Insert Figure 2>

We also observe from Figure 2 that, before the momentum crashes, the WML return usually spikes upward. The WML strategy cumulates positive returns, particularly through a negative beta loading in a bear market. As market volatility tends to increase in a bear market, the increase in WML profits is like an increase in the premium of a short-call position as market volatility increases (Daniel and Moskowitz, 2016). Greater market volatility suggests a higher chance of being in-the-money for a short-call position, indicating greater crash risk. Figure 3, Panel A, is an x-y plot of WML returns (y-axis) and current market returns (x-axis), which show that a large negative WML return is associated with positive current market returns. This resembles a payoff to a short-call option where a large negative return occurs when the option is in-the-money. Figure 3, Panel B is an x-y plot where the x-axis is past 12-month cumulative market returns, which shows that large negative WML returns tend to occur when the market return has been negative for the past 12 months. Thus, both panels together show that a momentum crash is likely to happen when the market rebounds after a bear market.

<Insert Figure 3>

Daniel and Moskowitz (2016) suggest that crashes are mostly attributable to the short side of the WML strategy, or loser portfolios that rebound strongly after a bear market. In an earlier study, Grundy and Martin (2001) show that time-varying beta generates negative beta exposure to the momentum strategy. They show that in a bear market, the loser portfolio will likely include high market beta assets, whereas the winner portfolio includes assets with low market beta. That is why the beta of the momentum strategy is likely to be negative following a bear market, exposing the strategy to reversal risk when the market recovers.

We point to the fact that, as the coskewness of the winner portfolio is more negative than that of the loser portfolio (Harvey and Siddique, 2000), the WML strategy would exhibit negative coskewness and is exposed to inherent downside risk. The coskewness of the WML portfolio returns, CS_{WML} is:

$$CS_{WML} = \frac{\text{cov}(r_W, r_m^2) - \text{cov}(r_L, r_m^2)}{\sigma_{WML}\sigma_m^2} = \frac{\sigma_W}{\sigma_{WML}} CS_W - \frac{\sigma_L}{\sigma_{WML}} CS_L \ , \quad (2)$$

where $\sigma^2_{WML} = \sigma^2_W + \sigma^2_L - 2\rho_{W,L}\sigma_W\sigma_L$. Thus, the coskewness of WML portfolio returns is not only determined by the coskewness of winner and loser portfolios, CS_W and CS_L, but also by the correlation between the two portfolios, $\rho_{W,L}$, which determine the volatility of WML returns, σ^2_{WML}. Figure 4, Panel A shows the time-series of the coskewness of 10 momentum portfolio returns. We observe occasional downward shifts of the coskewness range, possibly due to sudden and large market movements that are associated with negative portfolio returns. Notably, many of the large downward shifts of the coskewness range occur outside periods of momentum crashes that are indicated by the shaded areas in the figure. As we compute the coskewness using an expanding window, adding one observation each month, coskewness shows a more persistent pattern towards the latter half of the observation period. However, what matters here is the difference in coskewness between the winner and loser portfolios, not the fluctuation of the level of coskewness, as we show below.

Figure 4, Panel B, shows the difference in coskewness of winner and loser portfolios (the coskewness spread of P10 and P1) and the coskewness of WML returns, CS_{WML}, where we find that short-term fluctuations of CS_{WML} and the CS_W-CS_L spread move in tandem except around 2000 when CS_{WML} shifts upward. We find that the shift is due to the increase in σ_{WML}, which is caused by the decrease in the (positive) correlation of the winner and loser portfolio returns, $\rho_{W,L}$.[7] Importantly, we find that the coskewness of WML portfolio returns tends to become less negative during periods with high volatility in a bear market, ahead of momentum crashes. During such periods, because market volatility becomes associated with higher WML returns, the coskewness of WML returns becomes less negative as the market comes closer to a bottom. As momentum crashes tend to occur after bear markets (Figure 2), a less negative coskewness not only signals low future returns but also a market bottom. We observe substantial downward shifts in both CS_{WML} and CS_W-CS_L during the momentum crashes from 2000 to 2009, which are manifestations of downside risk. However, such downward shifts are not obvious for the momentum crashes in 2016 and from 2018 to 2019. Therefore, we further explore the dynamics of the coskewness using a time series regression.

<Insert Figure 4>

We examine the time-series properties of CS_{WML} with a monthly regression where the dependent variable is the change in the coskewness, $CS_{WML,t} - CS_{WML,t-1}$. We use the difference in coskewness between the current and previous month since the variable is highly persistent.[8] The independent variables are lagged coskewness, $CS_{WML,t-1}$, an indicator variable $I_{Bear,t-1}$ which equals one if the cumulative market return is negative for the prior 12 months or zero otherwise, and the market volatility, $Mvol_{t-1}$, which is measured by the standard deviation of daily market return in the previous six months. We also include the contemporaneous market return over the risk-free rate, $r_{m,t}$. Thus, we have

[7] We examined the correlation using all observations up to each month.

[8] The p-value of the Dicky-Fuller unit-root test for CS_{WML} is 0.3375, and that of the difference in CS_{WML} is <0.0001.

$$CS_{WML,t} - CS_{WML,t-1} = a_0 + b_B I_{Bear,t-1} + (b_M + b_{B,M} I_{Bear,t-1}) r_{m,t} + b_C CS_{WML,t-1} + b_V Mvol_{t-1} + e_t, \quad (3)$$

where b_M and $b_{B,M}$ indicate the relation between the coskewness of WML and the market return following a bull and bear market period, respectively. We also control for the effects of lagged coskewness and lagged market volatility. The regression results are in the first column (CS_{WML}) of Table 5. We find that the coefficients b_M, $b_{B,M}$, and b_C are statistically significant. A positive coefficient b_M suggests that CS_{WML} tends to comove with the market return after a bull market, indicating the coskewness increases and becomes less negative when the market return is positive (e.g., during the 1990s). On the other hand, the negative coefficient of the interaction term between I_{Bear} and r_m, $b_{B,M}$, not only reflects the increase in CS_{WML} when the bear market prolongs (r_m continues to be negative) but also a large drop of CS_{WML} when the market rebounds (r_m>0), after a bear market and when a momentum crash might occur. Thus, the regression result indicates an increase in CS_{WML} when the bull market continues, a further increase in CS_{WML} in a bear market as the market approaches the bottom, and a large reversal when the market rebounds.

<Insert Table 5>

In columns CS_W and CS_L of Table 5, we report the coskewness of the winner and loser portfolios separately to examine the different dynamics of the long and short legs of the WML return. To do so, we replace the CS_{WML} in equation (3) with CS_W and CS_L, respectively. For both CS_W and CS_L, we find that the coefficient b_B is positive and significant, indicating that the coskewness of both portfolios tends to increase and becomes less negative after bear markets. This is likely because negative returns become less associated with market volatility as the market approaches the bottom. Next, the b_M coefficient is positive and significant for both regressions, implying an increase in the coskewness of both winner and loser portfolios following bull markets and when the market return is positive. The coefficient $b_{B,M}$ is significantly negative for the winner portfolio, whereas it is negative and insignificant for the loser portfolio. The result shows that the coskewness of the winner portfolio further increases following a bear market and when the current market return continues to be negative, whereas that of the loser portfolio does not. Thus, it is the winner portfolio that mainly explains the increase in CS_{WML} after a bear market. During a momentum crash as the market rebounds, the winner portfolio also explains the large drop of CS_{WML} as its coefficient $b_{B,M}$ is negative and statistically significant.

Our next regression, Equation (4), examines if lag CS_{WML} predicts monthly WML returns, r_{WML}, and investigates the dynamic predictive effect of CS_{WML} on r_{WML}. The main independent variables are the *l*-month (*l*=1, 3, 6, 9, and 12) lagged coskewness of WML returns, $CS_{WML,t-l}$. Other independent variables include a dummy variable for the bear market from month *t-12* to *t-1*, I_{Bear}; a dummy variable for positive market return in month *t*, $I_{PosMkt,t}$; and the standard deviation of daily market return through month *t-6* to *t-1*, $Mvol_{t-1}$. We also interact CS_{WML} with I_{Bear} and I_{PosMkt} to examine if the impact of coskewness on WML return differs when the market rebounds after a period of a bear market. We perform a pair of regressions for each lagged coskewness; a simple regression with CS_{WML} and the fully specified regression as shown below.

$$r_{WML,t} = a + b_0 r_{m,t-1} + b_{B,M} I_{Bear,t-1} I_{PosMkt,t} + (c_0 + c_{B,M} I_{Bear,t-1} I_{PosMkt,t}) CS_{WML,t-l} + vMvol_{t-1} + e_t \quad (4)$$

From Table 6, for all lagged coskewness, the simple regressions show that the coefficients of the coskewness are negative and statistically significant at the 1% level, indicating that lagged coskewness significantly predicts future WML returns. In the full model, the coefficients of r_m and *Mvol* in all regressions are both negative and statistically significant, confirming the previous empirical findings that the momentum strategy has a negative beta and does not perform well in volatile markets. The coefficient of the product of $I_{Bear,t-1}$ and $I_{PosMkt,t}$, $b_{B,M}$, is significantly negative at the 1% significance level, which also confirms that WML returns tend to crash when the stock market rebounds following a bear market. For the interactive term between CS_{WML} and the two dummy variables, its coefficients, $c_{B,M}$, are negative and strongly significant for all regressions, with the magnitudes of the coefficients decreasing from -33.45 (l=1) to -22.81 (l=12) for longer lags. This result suggests that the negative effect of lagged coskewness on WML is remarkably stronger in times when the stock market rebounds from a bear market. Taking the 1-month lagged coskewness as an example, whose impact on WML return is -35.90 (c_0+c_{BM}), for periods when the market rebounds after a bear market, this number translates into a -3.08% decrease in next-month WML return for a one-standard-deviation increase in coskewness (-35.90×0.0857). As momentum crashes tend to last over several months, by using the coefficients of coskewness for l=1, 3, and 6, the estimated impact of a one-standard-deviation increase in coskewness on WML return is around -15.89% over the next six months.[9] Our result indicates that the past increase of coskewness predicts a momentum reversal when the market rebounds after a bear market. Since the interactive term includes a contemporaneous term, $I_{PosMkt,t}$, the coefficient of coskewness for the rest of the sample periods (c_0) is subsumed by the interactive term.[10] To sum up, Table 6 shows that lagged CS_{WML} can negatively predict WML returns, and such an effect is particularly stronger when momentum crashes might occur after a bear market.

<Insert Table 6>

[9] We estimate the impact of a one standard deviation increase in CS_{WML} on the WML return over the next six months by assuming the regression coefficients of the 1-, 3-, and 6-month lagged CS_{WML} are roughly the same as for the 2-, 4-, and 5-month lagged CS_{WML}, respectively. Therefore, the estimated impact is calculated as (-35.90-31.98-24.85)×2.

[10] In unreported regression results, if we use $I_{PosMkt,t-1}$ instead of $I_{PosMkt,t}$, the average impact of coskewness, c_0, is negative and statistically significant, suggesting that the contemporaneous market condition has significant impact on the coefficient.

5. The coskewness augmented momentum trading strategy

Although Grundy and Martin (2001) imply that using betas to rebalance momentum portfolios might hedge momentum reversals, Daniel and Moskowitz (2016) suggest that such hedging might not be practically feasible since such hedging relies on future realized betas. Alternatively, they show a hedging strategy that adjusts the exposure using the predicted Sharpe ratio of the WML portfolio. In their paper, a low Sharpe ratio reflects downside risk after bear markets when expected WML portfolio returns are lower and their volatility greater. Barroso and Santa-Clara (2015) propose a strategy that manages the volatility of WML returns by directly using the predicted volatility of the WML portfolio returns. Specifically, maintaining constant volatility, their strategy reduces the exposure to the baseline WML strategy when standard deviation increases, and vice versa. Their strategy is based on the finding that the realized standard deviation of WML returns could predict future momentum reversals.

Our time-series regressions in the previous section show that the coskewness of WML portfolio returns predicts future WML returns, and such an effect is remarkably amplified around times when momentum crashes are more likely to occur. Thus, we propose a trading strategy that reduces the exposure to the WML strategy when coskewness is above a certain threshold and increases it when coskewness is below that threshold. We augment the volatility-managed BSC strategy with a coskewness-managed strategy to improve the downside property. The original BSC strategy significantly improves the performance of the baseline momentum strategy by increasing the average return and lowering the standard deviation. Also, the BSC strategy reduces the left skewness of momentum returns to a certain degree. However, the BSC strategy still yields a negative skewness, suggesting that the left tail is longer than the right one. With their method, the momentum crash risk, although tempered to some extent, still exists. In contrast, we propose to use the coskewness to further alleviate the crash risk of the momentum strategy.

Since our coskewness augmented strategy increases the exposure to the WML portfolio when the WML coskewness becomes more negative and vice versa, the exposure is a decreasing function of the coskewness. In this way, our strategy reduces the exposure to a future reversal when the coskewness becomes less negative before a reversal. Since a less negative coskewness of momentum return signals lower future returns, our strategy partially hedges low future WML returns. When the coskewness is more negative, our strategy increases the exposure and takes advantage of high future returns.

Since the momentum strategy is a zero-cost strategy in which the long and short positions have the same dollar value, we can scale the weights up or down to a baseline \$1 investment in the WML strategy. Our proposed method (CS-BSC hereafter) utilizes coskewness to augment the standard deviation-based method to control the exposure to the baseline momentum strategy. Specifically, the weight to the baseline WML return in month *t* is

$$\left(1 + \frac{CS_{WML,t-1}{}^{*} - CS_{WML,t-1}}{Range}\right) * \left(\frac{\sigma_{WML,t-1}{}^{*}}{\sigma_{WML,t-1}}\right), \tag{5}$$

where $CS_{WML,t-1}$ is the coskewness of the WML return estimated with monthly returns up to month *t-1*, *Range* is the difference between the maximum and minimum CS_{WML} values during the estimation period, and $\sigma_{WML,t-1}$ is the standard deviation of WML returns estimated using daily returns in the previous six months. $CS_{WML,t-1}{}^{*}$ and $\sigma_{WML,t-1}{}^{*}$ represent the target coskewness and standard deviation values, respectively. We set the target $CS_{WML}{}^{*}$ and $\sigma_{WML}{}^{*}$ values as the rolling average coskewness and standard deviation calculated using historical data up to month *t-1*.[11]

[11] Alternatively, we also examined strategies with constant $CS_{WML}{}^{*}$ and $\sigma_{WML}{}^{*}$ targets, following Barroso and Santa-Clara (2015). Please refer to Appendix A1 for the details. The resulting BSC and CS-BSC weights are both higher

The proposed weight is a product of two components: a coskewness component and a standard deviation component. The standard deviation component is the same as the BSC model. The coskewness component is also a decreasing function of CS_{WML}, that is, the weight assigned to the WML strategy reduces as the CS_{WML} value increases. For comparison purposes, we also report the CS-only strategy, which uses only the first component in equation (5) as the weight function.

Figure 5, Panel A, shows the time series of the weights for the BSC and CS-BSC methods, and Panel B shows the difference between these weights. The weights from both methods fluctuate within the range of (0, 3.5), and they both reach their lows during the momentum crash periods. The average weights of the BSC and CS-BSC strategies across the sample period are 1.08 and 1.00, respectively, suggesting that the former slightly overweights the baseline strategy, and the latter has the same average exposure as the baseline strategy. The weight of the proposed method mostly moves in the same direction as the BSC method since they share the same standard deviation component. However, our method always underweights more than the BSC method during some of the biggest momentum crashes. Also, our method overweights more than the BSC method before the 2000s when the crash risk was lower than in later years.

<Insert Figure 5>

Table 7 compares the performances of the momentum strategies. We find that the CS-only strategy effectively enhances the performance of the baseline WML strategy, in terms of increasing the Sharpe ratio, Sortino ratio, and skewness and reducing the maximum drawdown. However, the CS-only strategy alone does not perform as well as the BSC strategy. For an investment with a 1-month holding period, the BSC method improves the baseline WML strategy by raising the Sharpe ratio from 0.83 to 1.47 and by lifting skewness from -2.58 to -0.65. This enhancement is consistent

than those calculated with rolling targets, but the strategy performances are close to those tabulated in Table 7. We thank an anonymous referee for suggesting the use of the rolling targets to avoid look-ahead bias.

with Barroso and Santa-Clara (2015). Therefore, we propose to use the coskewness of the WML return to enhance the BSC strategy to generate superior performance and further reduce the downside risk.

With the CS-BSC method, the Sharpe ratio increases slightly to 1.56. However, the Sortino ratio, which measures the relative performance against the downside volatility, improves substantially for the CS-BSC method compared with the BSC method: from 2.81 for the BSC method to 3.56 for the CS-BSC method. Moreover, the skewness shifts from negative to positive 0.21, indicating that the method has effectively mitigated the momentum crash risk and introduced a chance of more positive returns. The downside risk reduces as reflected in the maximum drawdown measure, which is -76% for the baseline WML strategy, -38% for the BSC method, and -30% for the CS-BSC method (i.e., a 61% reduction from the baseline WML strategy and a 20% reduction from CS-BSC strategy).[12]

<Insert Table 7>

Figure 6 shows the cumulative payoffs of the three strategies. Panel A adjusts the two risk-managed strategies to have the same volatility as the baseline WML strategy, while Panel B adjusts the two risk-managed strategies to have the same downside volatility as the baseline WML strategy. In both panels, the CS-BSC strategy outperforms the baseline and BSC strategies, with Panel B showing greater outperformance. This suggests that, when considering only the downside volatility, the proposed strategy offers a better risk-adjusted payoff due to its ability to reduce downside risk. Figure 6 also shows that the cumulative payoff of the CS-BSC strategy has the smallest downturn compared to the other two strategies during several momentum crash episodes.

<Insert Figure 6>

[12] To ensure comparability among the three strategies, we adjust BSC and CS-BSC strategies to have the same standard deviation as the baseline WML strategy for the maximum drawdown calculation, performance plot, and density plot.

Figure 7 plots the density of the monthly returns for the three strategies. Both risk-managed strategies have thinner and shorter tails on the negative side than the baseline strategy. Nonetheless, the coskewness augmented strategy has the shortest left tail (-21.95%, -14.90%, and -12.19% at the 1st percentile for WML, BSC, and CS-BSC, respectively), suggesting that this strategy has the lowest crash risk. Moreover, the CS-BSC strategy has an obvious longer tail on the right side than the BSC strategy (14.05%, 18.32%, and 20.22% at the 99th percentile for WML, BSC, and CS-BSC, respectively), which is consistent with the relatively higher positive skewness of the CS-BSC strategy returns.

<Insert Figure 7>

Since it is a widespread practice for momentum investors to rebalance the portfolio less frequently than monthly (e.g., the S&P 500 Momentum Index rebalances weights semi-annually), we further examine the performance of the proposed strategy for longer holding periods. Table 7 presents the baseline and enhanced momentum strategies' overlapping returns over 3, 6, 9, and 12 months.[13] For longer holding periods from 3 months to 12 months, we generally find a decrease in the mean return, standard deviation, and Sharpe ratio across the three WML strategies.[14] We find only a slight increase in the Sharpe ratio of the CS-BSC strategy compared with that of the BSC strategy for all holding periods. However, for longer holding periods, we find that our strategy brings greater improvements in mitigating downside risk relative to the baseline and BSC strategies, particularly for the 3- and 9-month holding periods. In general, the Sortino ratio, skewness, and the maximum drawdown improve compared to those of the BSC strategy, for longer

[13] The overlapping returns over H months are the returns of the long-short WML portfolio which replaces only the 1/H most stale constituent stocks with the latest winner and loser stocks each month, as defined in Jegadeesh and Titman (1993).

[14] Harvey and Siddique (2000) also show that mean return and volatility decreases as holding period increases. They also show that skewness and kurtosis decrease with holding periods, suggesting a trade-off between mean return and skewness.

holding periods. The improvement in the Sortino ratio of the CS-BSC strategy is the largest (29.1%) for the 3-month holding period from the BSC strategy. The skewness has the largest positive shifts of 0.94 and 0.93 from the BSC strategy for the 3- and 6-month-holding periods, respectively. The maximum drawdown has the largest improvements of 39% and 38% from the BSC strategy for the 6- and 9-month-holding periods.

Since both the BSC and CS-BSC methods use dynamic weights to adjust the exposure of the baseline WML strategy, the improved performance of the dynamic strategy might be used to balance the potential trading costs. Novy-Marx and Velikov (2016) show that the momentum strategy is profitable after their estimated trading cost, which is 0.65% per month. Barroso and Santa-Clara (2015) estimate that the turnover of the baseline momentum and BSC strategies are close. Barroso and Santa-Clara argue that given the superior performance of the dynamic trading strategy, trading costs should be a minor issue. Since the weight of the proposed CS-BSC strategy varies within a similar range to the BSC strategy, the concern that the trading cost could wipe out its performance enhancement effect could be alleviated.[15] More importantly, since the trading cost becomes smaller as the rebalancing frequency decreases, the CS-BSC strategy's after-cost performance would remain strong for longer holding periods.

Lastly, we examine how the strategies perform in various subperiods. We divide the whole sample period into three subperiods: the first subperiod (1974-1990) covers the 1987 crash, the second subperiod (1991-2004) covers the 2001 dot-com crash, and the third subperiod (2004-2019) covers the 2008-2009 global financial crisis. The details are presented in Table A2 in the appendix.

[15] With a back-of-the-envelope calculation, we estimate the after-cost strategy performance. During the sample period, the average weights of the BSC and CS-BSC strategies are 1.08 and 1.00, respectively. Using the cost data in Novy-Marx (2016), the trading costs for the BSC and CS-BSC strategies are roughly 0.70% and 0.65% per month, respectively. Applying these costs to the gross returns, the after-cost Sharpe (Sortino) ratios of the baseline WML, BSC and CS-BSC strategies are 0.46, 0.93, and 1.03 (0.65, 1.78, and 2.36), compared with 0.83, 1.47 and 1.56 (1.18, 2.81, and 3.56) without trading costs, respectively, for the one-month holding horizon. This suggests that the such cost increases do not offset the performance enhancement effect of the CS-BSC strategy.

Table A2 shows that the CS-BSC strategy offers effective downside risk reduction in all three subperiods, as it has the highest Sortino ratio and skewness and the smallest maximum drawdown. The results further confirm that the efficacy of the proposed CS-BSC strategy is robust over time.

In summary, our proposed momentum strategy effectively mitigates the downside risk for up to 12-month holding periods by controlling for coskewness and therefore improving the BSC volatility-based strategy. Our strategy not only generates a return distribution that is slightly positively skewed but is also effective in mitigating downside risk for longer holding periods.

6. International evidence

We now explore the international markets of the UK, France, Germany, and Japan using a sample from Datastream for January 1986 to December 2019. Our analysis incorporates all stocks traded on the London Stock Exchange (LSE), Tokyo Stock Exchange (TSE) First Section, Frankfurt Stock Exchange (FWB), and Paris Bourse (Euronext Paris from 2000 onwards).[16] We focus on these markets because they are the largest developed markets examined by Barroso and Santa-Clara (2015). Following their analysis, we convert all returns to US Dollars and use the US risk-free rate.

Table 8 reports the prior 11-months returns, stock level coskewness, coskewness of momentum portfolio returns, and the post-ranking value-weighted returns for 10 momentum portfolios in each market. For the UK, French and German markets, we find that the coskewness of individual stock returns and that of momentum portfolio returns are all negative. Moreover, the winner portfolios tend to have more negative coskewness than loser portfolios. We also find that

[16] As we use the first one year to calculate PR and the next 10 years to estimate CS, all our estimations are for 1997-2019.

the post-ranking portfolio returns in the last column maintain the same order as the ranking portfolios for the UK, French, and German markets. Furthermore, there exist statistically significant WML return spreads (at the bottom of the last column) for these markets, indicating a momentum effect.

For the Japanese market, we find that the coskewness of individual stock returns and momentum portfolio returns are both positive for all portfolios. We also find that the post-ranking portfolio returns fail to maintain the same order as the ranking portfolio returns. The WML return spread is not significantly different from zero, which confirms the lack of a momentum effect in the Japanese market as documented by previous studies (Chui et al., 2010; Fama and French, 2012; Asness et al., 2013).

<Insert Table 8>

In Table 9, we show the results of the FM regression for individual stock returns 1, 3, 6, 9, and 12 months forward. For the coefficients for prior 11-month returns, PR, we find that the coefficient is most significant for 1-month forward returns and becomes less significant for more future returns for the French and German markets, whereas for the UK market they are not significant for any forward return. The coefficient of PR for Japan is negative and even significant up to 12 months forward.

<Insert Table 9>

For the coefficient of CS_PORT in the UK, France, and Germany, we find that the coefficient tends to be more statistically significant for 3- to 9-month forward returns rather than for 1-month forward returns. For Japan, the coefficient is most significant for 1-month returns. Finally, the coefficient of individual stock return coskewness, CS_STK, is statistically non-significant for French stocks (except for F=12), and for the UK and Japanese stocks. However, we find that the CS_STK coefficients are statistically more significant than those of CS_PORT for German stocks.

As Table 8 shows that the coskewness of the winner portfolio is less than that of the loser portfolio, the coskewness of the WML strategy is negative for all international markets. Thus, we examine if our coskewness augmented strategy also works in these markets. We present the results for the three trading strategies in Table 10. Comparing the baseline WML and BSC strategies, we confirm that the latter significantly improves the Sharpe and Sortino ratios compared to the baseline momentum strategy for all holding periods (up to 12 months) for the UK, German, and French markets. We find that the skewness of the CS-BSC strategy shifts to the right compared with that of the BSC strategy for all holding periods and all markets, excluding Japan. The improvement of the maximum drawdown for the CS-BSC strategy over the BSC strategy is more evident for the German and French markets for all holding periods, with relatively less improvement for the UK market. Thus, we find that our coskewness augmented strategy generally improves the downside risk of the BSC strategy for the three international markets but not for Japan.

<Insert Table 10>

Figure 8 compares the cumulative payoffs and weights of the BSC and CS-BSC strategies for the four international markets. Like our result for the US market, the weight of the proposed method mostly moves in the same direction as the two methods share the same standard deviation component. Again, the CS-BSC method always underweights more than the BSC method during the biggest momentum crashes. In the long run, the CS-BSC method performs better than the BSC strategy in the UK, Germany, and France when the WML strategy experiences large crashes. However, as documented by previous studies, the Japanese market has been anomalous for momentum effects. Although the constant volatility BSC strategy might work for the Japanese market, our coskewness algorithm does not seem to further mitigate the downside risk for this market.

<Insert Figure 8>

7. Conclusion

The winner-minus-loser momentum strategy has an inherent downside risk, as the portfolio returns of the strategy always have a negative coskewness. The momentum strategy tends to cumulate profits in bear markets as the strategy displays a negative beta. We find that the coskewness of momentum portfolio returns tends to increase before a subsequent market rebound when momentum crashes are likely to happen. This is because market volatility becomes associated with fewer negative returns before a market rebound. We also find that the coskewness of momentum portfolio returns predicts individual stock returns; more negative coskewness predicts higher returns. Using this property of the coskewness of momentum portfolio returns, we propose a momentum strategy that augments the constant-volatility strategy of Barroso and Santa-Clara (2015). Our proposed strategy significantly mitigates the downside risk of the baseline momentum strategy as well as the constant-volatility strategy. We find that our augmented strategy not only mitigates the downside risk but also its mitigation effect improves for holding periods that are longer than one month. This is because the coskewness of momentum portfolio returns significantly predicts future stock returns up to nine months ahead, which supports the return of our momentum strategy beyond horizons for which past returns display predictive ability.

References

Amihud, Y., 2002. Illiquidity and stock returns: cross-section and time-series effects. Journal of Financial Markets, 5, 31-56.

Asness, C., Moskowitz, T., Pedersen, L., 2013. Value and momentum everywhere. Journal of Finance 68, 929-985.

Avramov, D., Chordia, T., Jostova, G., Philipov, A., 2007. Momentum and credit rating. Journal of Finance 62, 2503-2520.

Barroso, P., Santa-Clara, P., 2015. Momentum has its moments. Journal of Financial Economics 116, 111-120.

Chang, B., Christoffersen, P., Jacobs, K. 2013. Market skewness risk and the cross section of stock returns. Journal of Financial Economics 107, 46-68.

Chui, A.. Titman, S. Wei, K., 2010. Individualism and momentum around the world. Journal of Finance 65, 361-392.

Cooper, M., Gutierrez Jr, R., Hameed, A., 2004. Market states and momentum. Journal of Finance 59, 1345-1365.

Daniel, K., Moskowitz, T.J., 2016. Momentum crashes. Journal of Financial Economics, 122, 221-247.

Daniel, K., Jagannathan, R., Kim, S., 2019. A hidden Markov model of momentum. Working paper.

Ehsani, S. and Linnainmaa, J.T., 2019. Factor momentum and the momentum factor (No. w25551). National Bureau of Economic Research.

Fama, E., French, K., 1992. The cross-section of expected stock returns. Journal of Finance 47, 427-465.

———, 1996. The CAPM is wanted, dead or alive. Journal of Finance 51, 47-1958.

———, 2012. Size, value, and momentum in international stock returns. Journal of Financial Economics 105, 457-472.

———, 2015. A five-factor asset pricing model. Journal of Financial Economics 116, 1-22.

Fama, E.F., MacBeth, J.D., 1973. Risk, return, and equilibrium: Empirical tests. Journal of Political Economy 71, 607-636.

Griffin, J.M., Ji, X., Martin, J.S., 2005. Global momentum strategies. Journal of Portfolio Management 31, 3-39.

Grundy, B., Martin, J., 2001. Understanding the nature of the risks and the source of the rewards to momentum investing. Review of Financial Studies 14, 29-78.

Harvey, C., Siddique, A., 2000. Conditional skewness in asset pricing tests. Journal of Finance 55, 1263-1295.

Hong, H., Lim, T., Stein, J.C., 2000. Bad news travels slowly: Size, analyst coverage, and the profitability of momentum strategies. Journal of Finance 55, 265-295.

Hou, K., Xue, C., Zhang, L., 2020. Replicating anomalies. Review of Financial Studies 33, 2019-2133.

Jegadeesh, N., Titman, S., 1993. Returns to buying winners and selling losers: Implications for stock market efficiency. Journal of Finance 48, 65-91.

Jondeau, E., Zhang, Q., and Zhu, X. 2019. Average skewness matters. Journal of Financial Economics 134, 29-47.

Kelly, B.T., Moskowitz, T.J., Pruitt, S., 2021. Understanding momentum and reversal. Journal of Financial Economics 140, 726-743.

Kraus, A., Litzenberger, R. 1976. Skewness preference and the valuation of risk assets. Journal of Finance 31, 1085-1100.

———, 1983. On the distributional conditions for a consumption-oriented three moment CAPM. Journal of Finance 38, 1381-1391.

Luo, J., Subrahmanyam, A., Titman, S., 2021. Momentum and reversals when overconfident investors underestimate their competition. Review of Financial Studies 34, 351-393.

Menkhoff, L., Sarno, L., Schmeling, M., Schrimpf, A., 2012. Currency momentum strategies. Journal of Financial Economics 106, 660-684.

Moskowitz, T., Grinblatt, M., 1999. Do industries explain momentum? Journal of Finance 54(4), 1249-1290.

Moskowitz, T.J., Ooi, Y.H., Pedersen, L.H., 2012. Time series momentum. Journal of Financial Economics 104, 228-250.

Novy-Marx, R., 2015. Fundamentally, momentum is fundamental momentum (No. w20984). National Bureau of Economic Research.

Novy-Marx, R., Velikov, M., 2016. A taxonomy of anomalies and their trading costs. Review of Financial Studies, 29(1), 104-147.

Yang, X. and Zhang, H., 2019. Extreme absolute strength of stocks and performance of momentum strategies. Journal of Financial Markets 44, 71-90.

Table 1. Summary statistics of the decile momentum portfolios

This table reports the summary statistics of decile momentum portfolios that we formed based on the ranking of past return (PR), which is the cumulative raw return from t-12 to t-2 months. We form 100 size-momentum double sorted portfolios each month, calculate the post-ranking monthly value-weighted portfolio returns, and estimate the portfolio coskewness using the post-ranking returns of the portfolio. To compute the coskewness, we use an expanding window up to the previous month with at least 120-month observations. In Panel A, CS_PORT is the average coskewness of the stock's residing size-neutral momentum portfolio for each momentum decile which includes 10 portfolios. CS_STK is the coskewness estimated with the stock's monthly returns with an expanding window up to the previous month with at least 120-month observations. MV is the natural log of stock market value at the end of the previous month. BM is the book-to-market ratio calculated using the previous month's market value and the previous financial year's book equity. OP is the operating profit in million USD in the previous financial year. AG is the asset growth rate from the previous two years. MAX is the maximum daily return in % in the previous month. ILLIQ is the average daily Amihud (2002) ratio ($\times 10^2$) in the previous three months. The STD (in %) and SKEW are the standard deviation and skewness of the decile momentum portfolios calculated using monthly value-weighted portfolio returns in the full sample. The bottom row of P10-P1 (Winner-Loser) reports the difference between the winner and loser portfolios. The *t*-statistics of the difference are reported in parentheses. For STD, we report the *p*-value of the *F*-test of the significance of the difference in standard deviations between the winner and loser portfolios. For SKEW, we report the *p*-value of the Kolmogorov-Smirnov test for the difference between the distributions of winner and loser portfolio returns. Panel B presents the average returns in % of the value-weighted momentum portfolios in the 1st, 3rd, 6th, 9th, and 12th post-ranking months, respectively. The reporting period is from January 1974 to December 2019. *, **, and *** represent significance at the 10%, 5%, and 1% level, respectively.

Panel A. Statistics of the decile momentum portfolios

Portfolio	PR	CS_STK	CS_PORT	MV	BETA	BM	OP	AG	MAX	ILLIQ	STD	SKEW
P1 (Loser)	-0.40	-0.20	-0.19	5.76	1.27	0.87	0.33	0.15	9.64	336.67	8.33	1.18
P2	-0.24	-0.20	-0.21	5.73	1.19	0.88	0.30	0.13	7.43	357.71	6.06	0.45
P3	-0.14	-0.20	-0.26	5.69	1.14	0.90	0.19	0.12	6.60	336.93	5.35	0.27
P4	-0.07	-0.20	-0.28	5.67	1.10	0.91	0.25	0.11	6.05	318.81	4.93	-0.25
P5	0.00	-0.20	-0.32	5.61	1.07	0.94	0.32	0.10	5.76	311.53	4.67	-0.42
P6	0.07	-0.21	-0.37	5.56	1.05	0.95	0.29	0.11	5.53	286.61	4.53	-0.65
P7	0.15	-0.21	-0.38	5.51	1.04	0.96	0.32	0.11	5.42	275.75	4.53	-0.70
P8	0.24	-0.21	-0.42	5.46	1.05	0.97	0.32	0.11	5.46	262.59	4.57	-0.86
P9	0.39	-0.21	-0.44	5.39	1.08	1.01	0.48	0.11	5.73	257.08	4.90	-0.93
P10 (Winner)	0.94	-0.21	-0.44	5.25	1.18	1.06	0.58	0.14	6.78	226.73	6.16	-0.57
P10-P1	1.34***	-0.01	-0.26***	-0.50***	-0.10***	0.20***	0.25***	-0.01	-2.86***	-109.94***	-2.17	-1.75
t-statistic	(58.53)	(-1.14)	(-24.01)	(-8.97)	(-10.35)	(7.06)	(4.60)	(-1.53)	(-16.37)	(-7.51)	*p*-F test <0.001	*p*-KS test <0.001

Panel B. Average returns of the decile momentum portfolios in various post-ranking months

	Forward month				
Portfolio	F=1	F=3	F=6	F=9	F=12
P1 (Loser）	0.35	0.44	0.60	0.82	0.99
P2	0.67	0.73	0.74	0.94	1.00
P3	0.71	0.74	0.90	0.91	0.89
P4	0.80	0.84	0.89	1.00	0.91
P5	0.84	0.88	0.96	0.98	0.95
P6	0.80	0.95	0.96	0.94	0.86
P7	0.94	0.92	0.92	0.92	0.87
P8	0.96	0.98	0.95	0.91	0.88
P9	1.07	1.00	1.02	0.91	0.84
P10 (Winner)	1.15	1.02	0.93	0.81	0.73
P10-P1 (Winner-Loser)	0.79^{***}	0.58^{***}	0.33^{*}	-0.01	-0.26
t- statistic	(3.91)	(3.16)	(1.86)	(-0.05)	(-1.36)

Table 2. Stock level cross-sectional regression with coskewness and past return

This table reports the coefficients of Fama-MacBeth regressions across individual stocks. The dependent variable is the monthly return of individual stocks in the 1st, 3rd, 6th, 9th, and 12th post-ranking month respectively. All explanatory variables are measured in the current month. For the definition of explanatory variables, see Table 1. We report the averages of the coefficients estimated from the monthly cross-sectional regressions. All explanatory variables are standardized to have zero mean and unit variance. The Newey-West *t*-statistics are presented in parentheses. Adj_R^2 is the average of the adjusted R-squares of the month-by-month regressions. The reporting period is from January 1974 to December 2019. *, **, and *** represent significance at the 10%, 5%, and 1% level, respectively.

Forward month	Intercept	CS_STK	CS_PORT	PR	MV	BETA	BM	OP	AG	MAX	ILLIQ	Adj_R^2
F=1	0.7159***			0.2148***	-0.2343***	0.1119*	0.0709	1.4809***	-0.3434***	-0.5494***	0.0961**	4.90%
	(3.56)			(2.67)	(-3.62)	(1.65)	(1.47)	(4.95)	(-5.12)	(-11.42)	(2.54)	
	0.7576***	-0.0582**	-0.1658**	0.1609**	-0.2037***	0.1101*	0.0723	1.4916***	-0.3456***	-0.5398***	0.1004***	5.16%
	(3.22)	(-1.98)	(-1.99)	(2.42)	(-2.78)	(1.66)	(1.53)	(5.04)	(-5.22)	(-11.48)	(2.67)	
F=3	0.7382***			0.1513*	-0.0861	0.0615	0.1125*	1.2099***	-0.3526***	-0.1929***	0.0687*	4.61%
	(3.28)			(1.83)	(-1.34)	(0.86)	(1.91)	(4.25)	(-5.61)	(-3.50)	(1.93)	
	0.7588***	-0.0384	-0.2950***	0.0428	-0.0061	0.0665	0.1143**	1.2123***	-0.3554***	-0.1768***	0.0810**	4.85%
	(2.89)	(-1.24)	(-3.59)	(0.60)	(-0.08)	(0.93)	(1.98)	(4.24)	(-5.67)	(-3.30)	(2.29)	
F=6	0.8084***			0.0851	-0.0398	0.0557	0.1396**	0.9677***	-0.3558***	-0.1281**	0.0341	4.33%
	(3.61)			(1.19)	(-0.63)	(0.80)	(2.16)	(2.92)	(-5.56)	(-2.17)	(1.08)	
	0.8189***	-0.0305	-0.2675***	-0.0256	0.0319	0.0578	0.1362**	0.9697***	-0.3616***	-0.1125**	0.0445	4.55%
	(3.09)	(-1.03)	(-3.34)	(-0.44)	(0.45)	(0.84)	(2.13)	(2.91)	(-5.64)	(-1.98)	(1.40)	
F=9	0.8805***			-0.0293	-0.0459	0.0654	0.0655	0.7206**	-0.2740***	-0.0924*	0.0571	4.11%
	(3.88)			(-0.50)	(-0.71)	(0.96)	(0.94)	(2.41)	(-4.19)	(-1.70)	(1.64)	
	0.9597***	-0.0151	-0.1578**	-0.0782	-0.0195	0.0627	0.0625	0.7100**	-0.2771	-0.0877*	0.0624*	4.30%
	(3.65)	(-0.53)	(-1.96)	(-1.63)	(-0.27)	(0.93)	(0.90)	(2.36)	(-4.22)***	(-1.70)	(1.79)	
F=12	0.8401***			-0.0918	-0.0650	0.0690	0.0629	0.5366**	-0.2476***	-0.1596***	0.0525	4.00%
	(3.77)			(-1.60)	(-0.96)	(1.01)	(0.87)	(2.01)	(-3.89)	(-2.70)	(1.48)	
	0.8841***	-0.0156	-0.0376	-0.0954**	-0.0658	0.0665	0.0627	0.5260**	-0.2486***	-0.1592***	0.0536	4.17%
	(3.53)	(-0.52)	(-0.51)	(-2.02)	(-0.87)	(0.98)	(0.87)	(1.98)	(-3.93)	(-2.85)	(1.48)	

Table 3. Cross-sectional regression for the tail and non-tail stocks

This table reports the coefficients of Fama-MacBeth regressions across two groups of stocks. Panel A presents the regression for the tail stocks according to their past return (PR) rankings, which come from the top or bottom 10% of the sample. Panel B presents the regression results for the other stocks, for which the PR rankings fall in the middle 80% of the sample. The dependent variable is the monthly return of individual stocks in the 1st, 3rd, 6th, 9th, and 12th post-ranking months, respectively. See Table 1 for the definition of explanatory variables. We report the averages of the coefficients estimated from the monthly cross-sectional regressions. All explanatory variables are standardized to have zero mean and unit variance. The Newey-West *t*-statistics are presented in parentheses. Adj_R^2 is the average of the adjusted R-squares of the month-by-month regressions. The reporting period is from January 1974 to December 2019. *, **, and *** represent significance at the 10%, 5%, and 1% level, respectively.

Panel A. Tail stocks (top & bottom 10% momentum stocks)

Forward month	Intercept	CS_STK	CS_PORT	PR	MV	BETA	BM	OP	AG	MAX	ILLIQ	Adj_R^2
F=1	0.739***			0.177**	-0.299***	0.118	-0.029	2.696***	-0.473***	-0.654***	0.245	6.43%
	(3.13)			(2.39)	(-3.63)	(1.57)	(-0.44)	(4.13)	(-3.80)	(-8.48)	(1.38)	
	0.860***	-0.069	-0.357**	0.083	-0.198*	0.120	-0.032	2.634***	-0.483***	-0.642***	0.288	6.84%
	(2.93)	(-1.37)	(-2.43)	(1.30)	(-1.85)	(1.62)	(-0.48)	(4.03)	(-3.92)	(-8.51)	(1.58)	
F=3	0.633**			0.161**	-0.024	0.085	0.073	1.832***	-0.364***	-0.225***	0.141	5.89%
	(2.41)			(1.99)	(-0.31)	(1.09)	(0.91)	(2.96)	(-2.96)	(-2.70)	(0.75)	
	0.720**	-0.021	-0.474***	0.015	0.163*	0.090	0.076	1.827***	-0.378***	-0.190**	0.187	6.26%
	(2.23)	(-0.43)	(-3.33)	(0.21)	(1.68)	(1.17)	(0.96)	(2.96)	(-3.11)	(-2.34)	(1.01)	
F=6	0.726***			0.095	0.065	0.072	0.149	1.631**	-0.327***	-0.137*	0.341	5.53%
	(2.81)			(1.31)	(0.83)	(0.95)	(1.70)	(2.54)	(-2.83)	(-1.75)	(1.58)	
	0.751**	-0.013	-0.490***	-0.073	0.240**	0.074	0.134	1.517**	-0.342***	-0.109	0.354*	5.82%
	(2.42)	(-0.29)	(-3.65)	(-1.19)	(2.44)	(0.99)	(1.55)	(2.35)	(-2.96)	(-1.45)	(1.76)	
F=9	0.817***			-0.017	0.097	0.079	0.125	1.184*	-0.211*	-0.135*	0.248	5.23%
	(3.19)			(-0.29)	(1.31)	(1.14)	(1.29)	(1.86)	(-1.72)	(-1.79)	(1.16)	
	1.022**	0.000	-0.345***	-0.132**	0.200**	0.074	0.117	1.134*	-0.214*	-0.119	0.303	5.49%
	(3.39)	(-0.00)	(-2.86)	(-2.64)	(2.24)	(1.08)	(1.24)	(1.80)	(-1.72)	(-1.62)	(1.37)	
F=12	0.845***			-0.081	0.020	0.067	0.079	0.855	-0.415***	-0.191**	0.216	5.06%
	(3.20)			(-1.37)	(0.22)	(1.01)	(0.84)	(1.42)	(-3.00)	(-2.42)	(1.03)	
	0.904***	0.001	-0.036	-0.086	0.012	0.064	0.083	0.861	-0.400***	-0.194**	0.208	5.32%
	(3.09)	(0.01)	(-0.34)	(-1.63)	(0.11)	(0.95)	(0.87)	(1.42)	(-2.92)	(-2.49)	(0.99)	

Panel B. Middle stocks (middle 80% momentum stocks)

Forward month	Intercept	CS_STK	CS_PORT	PR	MV	BETA	BM	OP	AG	MAX	ILLIQ	Adj_R^2
F=1	0.677***			0.357***	-0.232***	0.112*	0.104**	1.605***	-0.297***	-0.507***	0.086**	4.81%
	(3.59)			(2.74)	(-3.67)	(1.72)	(2.04)	(3.96)	(-4.10)	(-10.78)	(2.16)	
	0.782***	-0.053*	0.109	0.456***	-0.273***	0.106	0.102**	1.569***	-0.303***	-0.505***	0.079	4.97%
	(3.82)	(-1.82)	(1.48)	(3.44)	(-3.99)	(1.64)	(2.01)	(3.97)	(-4.22)	(-10.72)	(2.05)	
F=3	0.767***			0.347***	-0.119*	0.067	0.130**	1.155***	-0.390***	-0.148***	0.071	4.52%
	(3.59)			(2.80)	(-1.87)	(0.95)	(2.27)	(3.26)	(-5.47)	(-2.82)	(2.06)	
	0.833***	-0.044	-0.063	0.313**	-0.100	0.064	0.132**	1.131***	-0.388***	-0.146***	0.073**	4.67%
	(3.58)	(-1.39)	(-0.92)	(2.47)	(-1.48)	(0.90)	(2.32)	(3.17)	(-5.45)	(-2.80)	(2.17)	
F=6	0.877***			0.298**	-0.094	0.063	0.127**	0.878**	-0.337***	-0.073	0.032	4.27%
	(4.23)			(2.60)	(-1.56)	(0.93)	(1.98)	(2.31)	(-4.25)	(-1.25)	(0.90)	
	0.903***	-0.030	-0.056	0.265**	-0.079	0.060	0.128**	0.891**	-0.339***	-0.071	0.032	4.41%
	(3.82)	(-1.01)	(-0.83)	(2.20)	(-1.13)	(0.88)	(2.00)	(2.34)	(-4.32)	(-1.23)	(0.90)	
F=9	0.900***			0.058	-0.092	0.063	0.028	0.820**	-0.313***	-0.038	0.059	4.14%
	(4.17)			(0.57)	(-1.42)	(0.96)	(0.41)	(2.26)	(-3.85)	(-0.65)	(1.52)	
	0.942***	-0.022	-0.071	0.037	-0.087	0.062	0.027	0.841**	-0.318***	-0.038	0.060	4.28%
	(3.92)	(-0.71)	(-1.06)	(0.34)	(-1.20)	(0.94)	(0.38)	(2.30)	(-3.92)	(-0.65)	(1.61)	
F=12	0.795***			-0.107	-0.087	0.070	0.051	0.511	-0.199**	-0.155***	0.041	4.00%
	(3.87)			(-0.99)	(-1.37)	(1.05)	(0.69)	(1.35)	(-2.60)	(-2.67)	(1.08)	
	0.751***	-0.010	-0.070	-0.143	-0.077	0.071	0.052	0.514	-0.203***	-0.155***	0.044	4.14%
	(3.39)	(-0.33)	(-1.10)	(-1.25)	(-1.08)	(1.06)	(0.70)	(1.39)	(-2.66)	(-2.72)	(1.14)	

Table 4. Cross-sectional regression for the most volatile and less volatile periods

This table reports the coefficients of Fama-MacBeth (1973) cross-sectional regressions across two groups of stocks. Panel A presents the regression results for the 20% most volatile market periods, using the 6-month daily market return volatility as the measure of market volatility. Panel B presents the regression results for the remaining 80% less volatile months. The dependent variable is the monthly return of individual stocks in the 1st, 3rd, 6th, 9th, and 12th post-ranking months, respectively. See Table 1 for the definition of explanatory variables. We report the averages of the coefficients estimated from the monthly cross-sectional regressions. All explanatory variables are standardized to have zero mean and unit variance. The *t*-statistics are presented in parentheses. Adj_R^2 is the average of the adjusted R^{2}s of the month-by-month regressions. The reporting period is from January 1974 to December 2019. *, **, and *** represent significance at the 10%, 5%, and 1% level, respectively.

Panel A. 20% Most volatile market periods

Forward month	Intercept	CS_STK	CS_PORT	PR	MV	BETA	BM	OP	AG	MAX	ILLIQ	Adj_R^2
F=1	1.293**			-0.378	-0.394**	0.502***	0.006	2.827***	-0.439***	-0.559***	0.011	6.27%
	(2.54)			(-1.40)	(-2.36)	(2.73)	(0.04)	(2.89)	(-2.68)	(-4.73)	(0.14)	
	1.737***	-0.200**	0.204	-0.252	-0.461**	0.486***	0.002	2.757***	-0.447***	-0.560***	0.005	6.72%
	(2.71)	(-2.45)	(0.72)	(-1.22)	(-2.25)	(2.73)	(0.02)	(2.86)	(-2.79)	(-4.95)	(0.06)	
F=3	1.059*			-0.394	-0.276**	0.306*	0.203	1.692**	-0.410***	-0.210*	0.072	5.54%
	(1.94)			(-1.53)	(-1.98)	(1.86)	(1.20)	(2.20)	(-3.01)	(-1.81)	(0.99)	
	1.197*	-0.127	-0.102	-0.402*	-0.235	0.315*	0.202	1.626**	-0.406***	-0.194*	0.082	5.88%
	(1.69)	(-1.56)	(-0.37)	(-1.96)	(-1.32)	(1.95)	(1.24)	(2.11)	(-2.96)	(-1.75)	(1.14)	
F=6	0.597			-0.251	-0.113	0.047	0.207	1.190	-0.416***	-0.228*	-0.022	4.77%
	(0.96)			(-1.32)	(-0.77)	(0.26)	(1.10)	(1.34)	(-3.75)	(-1.67)	(-0.37)	
	0.554	-0.049	-0.142	-0.326**	-0.075	0.056	0.195	1.141	-0.423***	-0.219*	-0.015	5.03%
	(0.71)	(-0.70)	(-0.58)	(-2.14)	(-0.43)	(0.32)	(1.06)	(1.28)	(-3.83)	(-1.67)	(-0.25)	
F=9	0.893			-0.412***	-0.111	0.239	0.139	0.335	-0.315*	-0.101	-0.035	4.86%
	(1.50)			(-2.67)	(-0.79)	(1.22)	(0.73)	(0.51)	(-1.68)	(-0.83)	(-0.50)	
	1.055	0.006	0.162	-0.344***	-0.165	0.236	0.127	0.291	-0.321*	-0.115	-0.040	5.07%
	(1.44)	(0.10)	(0.69)	(-2.82)	(-0.99)	(1.23)	(0.68)	(0.43)	(-1.71)	(-1.01)	(-0.57)	
F=12	1.164***			-0.293**	-0.228	0.350**	0.156	0.438	-0.166	0.028	0.021	4.87%
	(2.73)			(-2.38)	(-1.59)	(2.03)	(0.73)	(0.65)	(-1.51)	(0.17)	(0.29)	
	1.321**	-0.093	0.215	-0.173**	-0.319*	0.339**	0.148	0.421	-0.176	0.015	0.009	5.07%
	(2.43)	(-1.29)	(0.97)	(-2.16)	(-1.87)	(2.00)	(0.70)	(0.65)	(-1.57)	(0.10)	(0.11)	

Panel B. 80% less volatile months												
Forward month	Intercept	CS_STK	CS_PORT	PR	MV	BETA	BM	OP	AG	MAX	ILLIQ	Adj_R^2
F=1	0.534**			0.402***	-0.184***	-0.011	0.091*	1.057***	-0.313***	-0.546***	0.123***	4.47%
	(2.55)			(6.62)	(-2.75)	(-0.17)	(1.92)	(4.39)	(-4.38)	(-10.68)	(2.91)	
	0.449*	-0.013	-0.282***	0.291***	-0.123*	-0.008	0.094**	1.093***	-0.314***	-0.534***	0.131***	4.67%
	(1.93)	(-0.47)	(-4.12)	(5.10)	(-1.73)	(-0.12)	(1.99)	(4.56)	(-4.42)	(-10.55)	(3.13)	
F=3	0.637***			0.324***	-0.026	-0.016	0.084	1.057***	-0.334***	-0.188***	0.068*	4.32%
	(2.73)			(4.95)	(-0.39)	(-0.21)	(1.53)	(3.65)	(-4.95)	(-3.07)	(1.68)	
	0.620**	-0.010	-0.356***	0.183***	0.066	-0.012	0.086	1.082***	-0.339***	-0.171***	0.081**	4.53%
	(2.44)	(-0.33)	(-5.42)	(3.06)	(0.94)	(-0.16)	(1.58)	(3.71)	(-5.05)	(-2.85)	(2.01)	
F=6	0.876***			0.192***	-0.017	0.058	0.118**	0.897***	-0.337***	-0.096	0.052	4.18%
	(3.98)			(2.91)	(-0.26)	(0.79)	(2.03)	(3.07)	(-4.48)	(-1.54)	(1.44)	
	0.903***	-0.025	-0.308***	0.070	0.066	0.058	0.118**	0.915***	-0.342***	-0.079	0.064*	4.39%
	(3.68)	(-0.79)	(-4.62)	(1.28)	(0.95)	(0.79)	(2.02)	(3.09)	(-4.53)	(-1.30)	(1.77)	
F=9	0.876***			0.092	-0.025	0.010	0.042	0.843**	-0.261***	-0.090	0.086**	3.87%
	(3.96)			(1.52)	(-0.37)	(0.16)	(0.70)	(2.63)	(-3.82)	(-1.50)	(2.34)	
	0.929***	-0.022	-0.259***	0.006	0.027	0.008	0.042	0.843**	-0.263***	-0.079	0.095**	4.05%
	(3.82)	(-0.65)	(-3.82)	(0.11)	(0.36)	(0.12)	(0.69)	(2.62)	(-3.85)	(-1.38)	(2.58)	
F=12	0.739***			-0.029	-0.014	-0.018	0.034	0.567*	-0.273***	-0.218***	0.062*	3.72%
	(3.26)			(-0.51)	(-0.19)	(-0.29)	(0.53)	(1.97)	(-3.75)	(-3.43)	(1.70)	
	0.748***	0.008	-0.116*	-0.071	0.013	-0.018	0.036	0.559*	-0.271***	-0.213***	0.068*	3.89%
	(2.99)	(0.24)	(-1.80)	(-1.30)	(0.16)	(-0.29)	(0.56)	(1.93)	(-3.82)	(-3.47)	(1.82)	

Table 5. Dynamics of the coskewness of WML return

This table presents the results of the regression $CS_{i,t} - CS_{i,t-1} = a_0 + b_B I_{Bear,t-1} + (b_M + b_{B,M} I_{Bear,t-1}) r_{m,t} + b_C CS_{i,t-1} + b_V \mathrm{Mvol}_{t-1} + e_t$. The dependent variable is the difference between the current month and the previous month's coskewness of the winner-minus-loser (*i*=*WML*) strategy returns, winner portfolio returns (*i*=*W*), or loser portfolio returns (*i*=*L*); I_{Bear} is an indicator variable that equals one if the prior 12-month cumulative market return is negative for a specified period, or zero otherwise; *Mvol* is market volatility which is measured by the standard deviation of daily market returns in the previous six months; and *RM* is the market return in excess of the risk-free rate. The reporting period is from January 1974 to December 2019. The *t*-statistics are presented in parentheses. *, **, and *** represent significance at the 10%, 5%, and 1% level, respectively.

Coefficient	$CS_{WML,t} - CS_{WML,t-1}$		$CS_{W,t} - CS_{W,t-1}$		$CS_{L,t} - CS_{L,t-1}$	
	Estimate	*t*-statistic	Estimate	*t*-statistic	Estimate	*t*-statistic
a_0	-0.003	(-1.25)	-0.005*	(-1.81)	-0.005**	(-2.28)
b_B	0.001	(0.82)	0.006**	(2.57)	0.005**	(2.39)
b_M	0.086***	(5.14)	0.187***	(7.10)	0.118***	(4.91)
$b_{B,M}$	-0.194***	(-7.58)	-0.092**	(-2.28)	0.040	(1.07)
b_C	-0.014**	(-2.09)	-0.007	(-1.43)	-0.007	(-1.40)
b_V	-0.001	(-0.50)	-0.001	(-0.51)	0.001	(0.31)
Adj_R^2	9.35%		10.11%		9.95%	

Table 6. Coskewness of WML and future WML return

This table presents the results of the regression $r_{WML,t} = a + b_0 r_{m,t-1} + b_{B,M} I_{Bear,t-1} I_{PosMkt,t} + (c_0 + c_{B,M} I_{Bear,t-1} I_{PosMkt,t}) CS_{WML,t-l} + vMvol_{t-1} + e_t$. The dependent variable is the monthly return of the winner-minus-loser strategy; r_m is the monthly market return in excess of the risk-free rate; $CS_{WML,t-l}$ is the l-month lagged coskewness value of the WML return, where l=1, 3, 6, 9, and 12; and *Mvol* is the standard deviation of daily market return in the previous six months. I_{Bear} is an indicator variable that equals one if the prior 12-month cumulative market return is negative, or zero otherwise. I_{PosMkt} is an indicator variable that equals one if the contemporaneous month's market return is positive, or zero otherwise. The reporting period is from January 1974 to December 2019. The *t*-stats are presented in parentheses. *, **, and *** represent significance at the 10%, 5%, and 1% level, respectively.

Lag of CS	a	b_0	$b_{B,M}$	c_0	$c_{B,M}$	v	Adj_R^2
	-0.24			-9.26***			1.73%
l=1	(-0.39)			(-3.11)			
	2.86***	-0.21***	-10.27***	-2.45	-33.45***	-1.12*	15.68%
	(3.27)	(-3.88)	(-6.52)	(-0.80)	(-4.45)	(-1.88)	
	-0.29			-9.54***			1.83%
l=3	(-0.48)			(-3.20)			
	2.86***	-0.19***	-9.36***	-2.40	-29.58***	-1.13*	14.88%
	(3.26)	(-3.52)	(-6.04)	(-0.78)	(-3.93)	(-1.88)	
	-0.26			-9.33***			1.75%
l=6	(-0.43)			(-3.13)			
	3.06***	-0.18***	-7.89***	-2.15	-22.69***	-1.31**	13.56%
	(3.47)	(-3.42)	(-5.16)	(-0.69)	(-2.98)	(-2.17)	
	-0.21			-8.97***			1.44%
l=9	(-0.33)			(-3.01)			
	2.97***	-0.19***	-8.33***	-2.53	-24.55***	-1.29**	13.63%
	(3.42)	(-3.45)	(-5.09)	(-0.83)	(-3.02)	(-2.14)	
	-0.18			-8.77***			1.55%
l=12	(-0.29)			(-2.94)			
	2.87***	-0.18***	-8.22***	-3.16	-22.81***	-1.32**	13.45%
	(3.32)	(-3.33)	(-4.80)	(-1.04)	(-2.75)	(-2.19)	

Table 7. Performances of the WML, volatility- and coskewness-enhanced momentum trading strategies

This table presents the performances of the baseline momentum strategy (WML), the volatility strategy (BSC) proposed by Barroso and Santa-Clara (2015), and the coskewness enhanced volatility strategy (CS-BSC). For the CS-only strategy, the weight in month t is $(1 + (CS^* - CS_{t-1}/Range))$, where CS^* is the target coskewness, CS_{t-1} is the coskewness of the monthly WML return estimated with monthly returns up to month *t-1*, and *Range* is the difference between the maximum and minimum coskewness in the estimation period. For the BSC strategy, the weight is σ^*/σ_{t-1}, where σ^* is the target standard deviation and σ_t is the standard deviation of the momentum strategy calculated using 6-month daily returns. For the CS-BSC strategy, the weight is $(1 + (CS^* - CS_{t-1}/Range)) * (\sigma^*/\sigma_{t-1})$. σ^* and CS^* are set as the moving average of the monthly standard deviation and coskewness of WML, estimated with data up to month *t-1*. We show the performances for various holding periods ranging from 1 month to 12 months. The overlapping returns over H months are the returns of the long-short WML portfolio which replaces only the 1/H most stale constituent stocks with the latest winner and loser stocks each month, as defined in Jegadeesh and Titman (1993). AMEAN, ASTD, and Sharpe represent the annualized average percentage return, standard deviation, and Sharpe ratio. Sortino represents the Sortino ratio which is the average return divided by the downside standard deviation. SKEW is the skewness of the monthly return. MIN and MAX are the minimum and maximum monthly returns, respectively. Maximum Drawdown measures the maximum reduction of the cumulative payoff during the holding period for the three strategies which are scaled to have the same standard deviation as the baseline momentum strategy. The reporting period is from January 1974 to December 2019.

Holding Period	Strategy	AMEAN	ASTD	Sharpe	Sortino	SKEW	MIN	MAX	Maximum Drawdown
H=1	WML	17.45	20.98	0.83	1.18	-2.58	-51.78	20.76	-76%
	CS-only	17.74	16.71	1.06	1.69	-1.91	-45.34	18.74	-62%
	BSC	22.99	15.66	1.47	2.81	-0.65	-24.30	16.15	-38%
	CS-BSC	23.18	14.87	1.56	3.56	0.21	-21.28	19.09	-30%
H=3	WML	15.29	19.51	0.78	1.12	-2.36	-44.51	19.14	-74%
	CS-only	15.81	15.62	1.01	1.63	-1.48	-38.98	18.30	-61%
	BSC	20.65	14.77	1.40	2.66	-0.60	-20.89	15.79	-38%
	CS-BSC	21.03	14.05	1.50	3.43	0.34	-18.29	19.34	-25%
H=6	WML	12.4	17.9	0.69	0.99	-2.05	-36.47	16.77	-72%
	CS-only	13.00	14.44	0.90	1.46	-1.11	-31.93	17.89	-58%
	BSC	17.26	13.79	1.25	2.34	-0.54	-19.71	15.29	-43%
	CS-BSC	17.66	13.11	1.35	2.99	0.39	-14.99	20.18	-27%
H=9	WML	9.79	16.3	0.60	0.87	-1.73	-29.27	14.22	-69%
	CS-only	10.36	13.24	0.78	1.27	-0.81	-25.63	18.65	-54%
	BSC	14.23	12.82	1.11	2.04	-0.47	-19.06	15.94	-44%
	CS-BSC	14.53	12.14	1.20	2.55	0.43	-13.65	21.04	-27%
H=12	WML	7.27	15.11	0.48	0.69	-1.64	-23.1	13.69	-67%
	CS-only	7.85	12.24	0.64	1.02	-0.69	-20.23	18.08	-52%
	BSC	11.24	12.08	0.93	1.63	-0.58	-19.95	15.96	-43%
	CS-BSC	11.51	11.30	1.02	2.03	0.36	-15.23	20.40	-28%

Table 8. Momentum portfolios' statistics for international markets

This table represents the summary statistics of decile momentum portfolios for the UK, Germany, France, and Japan. See Table 1 for the definition of variables. The reporting period is from January 1997 to December 2019. *, **, and *** represent significance at the 10%, 5%, and 1% level, respectively.

Portfolio	PR	CS_STK	CS_PORT	VW	PR	CS_STK	CS_PORT	VW
	UK				Germany			
P1 (Loser)	-0.49	-0.15	-0.19	-0.15	-0.43	-0.20	-0.28	-0.27
P2	-0.30	-0.16	-0.23	0.03	-0.27	-0.21	-0.36	0.15
P3	-0.20	-0.16	-0.29	0.12	-0.17	-0.21	-0.36	0.21
P4	-0.11	-0.16	-0.33	0.44	-0.10	-0.21	-0.35	0.31
P5	-0.02	-0.16	-0.33	0.47	-0.02	-0.21	-0.36	0.44
P6	0.06	-0.16	-0.39	0.63	0.05	-0.21	-0.42	0.55
P7	0.16	-0.16	-0.40	0.69	0.13	-0.21	-0.44	0.75
P8	0.27	-0.16	-0.41	0.93	0.23	-0.20	-0.41	0.83
P9	0.45	-0.16	-0.44	1.02	0.39	-0.20	-0.43	0.99
P10 (Winner)	1.04	-0.15	-0.48	1.39	1.02	-0.19	-0.39	1.25
P10-P1	1.52***	0.01	-0.28***	1.55***	1.45***	0.02**	-0.11***	1.52***
t-statistic	(42.18)	(1.36)	(-64.58)	(2.82)	(34.28)	(2.58)	(-31.21)	(3.09)
	France				Japan			
P1 (Loser)	-0.42	-0.14	-0.07	0.09	-0.36	0.16	0.39	0.30
P2	-0.25	-0.14	-0.19	0.30	-0.21	0.18	0.39	0.36
P3	-0.15	-0.15	-0.18	0.55	-0.14	0.18	0.39	0.35
P4	-0.07	-0.15	-0.14	0.56	-0.08	0.18	0.37	0.37
P5	0.01	-0.15	-0.20	0.66	-0.02	0.19	0.37	0.45
P6	0.08	-0.15	-0.21	0.77	0.04	0.18	0.32	0.37
P7	0.16	-0.16	-0.22	0.87	0.11	0.18	0.32	0.49
P8	0.25	-0.15	-0.28	0.88	0.19	0.18	0.31	0.47
P9	0.41	-0.15	-0.29	1.10	0.33	0.16	0.27	0.39
P10 (Winner)	1.00	-0.14	-0.35	1.37	0.79	0.14	0.30	0.30
P10-P1	1.42***	0.00	-0.29***	1.28**	1.15***	-0.02***	-0.08***	0.00
t- statistic	(41.60)	(0.40)	(-41.52)	(2.48)	(33.36)	(-3.35)	(-20.99)	(0.00)

Table 9. International evidence: FM regression results

This table reports the coefficients of Fama-MacBeth regressions for the UK, German, French, and Japanese markets. All returns and financial data are converted to US dollars. The dependent variable is the monthly return of individual stocks in the 1st, 3rd, 6th, 9th, and 12th post-ranking months, respectively. See Table 1 for the definition of variables. We report the averages of the coefficients estimated from the monthly cross-sectional regressions. All explanatory variables are standardized to have zero mean and unit variance. The Newey-West *t*-statistics are presented in parentheses. Adj_R^2 is the average of the adjusted R-squares of the month-by-month regressions. The reporting period is from January 1997 to December 2019. *, **, and *** represent significance at the 10%, 5%, and 1% level, respectively.

Forward month	Intercept	CS_STK	CS_PORT	PR	MV	BETA	BM	OP	AG	MAX	ILLIQ	Adj_R^2
						UK						
F=1	0.0096	-0.0097	-0.1154*	0.1886	0.1084	-0.0268	0.4431	0.1375	1.2357	-0.0711	-0.2235**	3.25%
	(0.03)	(-0.24)	(-1.87)	(1.11)	(1.18)	(-0.39)	(1.01)	(0.71)	(0.62)	(-0.79)	(-2.13)	
F=3	-0.1917	0.0092	-0.1388**	0.2253	0.1728*	-0.0054	0.3076	0.0942	-2.6887	-0.4538***	-0.0594	3.14%
	(-0.55)	(0.22)	(-2.09)	(1.18)	(1.93)	(-0.07)	(0.68)	(0.48)	(-1.17)	(-4.80)	(-0.70)	
F=6	-0.2590	-0.0180	-0.1232**	0.2286	0.1946**	-0.0563	-0.0306	0.2682	-3.5689	-0.2428**	-0.1261	2.72%
	(-0.65)	(-0.35)	(-2.47)	(1.13)	(2.24)	(-0.75)	(-0.06)	(1.45)	(-1.77)	(-2.41)	(-1.42)	
F=9	-0.0433	0.0151	-0.1946***	0.1750	0.2908***	-0.0090	-0.1842	0.3193	-1.1468	-0.1915**	0.1097	2.40%
	(-0.10)	(0.29)	(-3.27)	(1.02)	(2.79)	(-0.12)	(-0.37)	(1.62)	(-0.67)	(-2.05)	(1.21)	
F=12	0.0254	0.0048	-0.1126	0.0791	0.2424**	-0.0442	-0.0693	0.4146*	2.4783	0.1080	-0.0578	2.29%
	(0.06)	(0.10)	(-1.54)	(0.53)	(2.19)	(-0.69)	(-0.13)	(1.90)	(1.40)	(0.89)	(-0.70)	
						Germany						
F=1	0.9062**	-0.2964***	-0.0356	1.5922***	-0.2437**	0.1937	0.8204	1.3920	1.3272	-1.1089***	1.7924*	6.39%
	(2.42)	(-5.00)	(-0.53)	(3.53)	(-2.42)	(0.99)	(0.80)	(1.52)	(0.22)	(-3.15)	(1.82)	
F=3	1.2715***	-0.2872***	-0.1269**	0.9823**	-0.1031	0.2255	0.2023	2.2996**	2.0221	-0.6764**	4.5609***	5.99%
	(2.93)	(-4.54)	(-1.97)	(2.01)	(-1.06)	(1.12)	(0.18)	(2.45)	(0.27)	(-1.95)	(2.92)	
F=6	0.5362	-0.2322***	-0.1120*	0.6620	-0.0796	0.0884	-0.2180	0.9102	-1.8589	-0.2419	1.1284	5.28%
	(1.24)	(-3.47)	(-1.66)	(1.18)	(-0.83)	(0.47)	(-0.25)	(1.00)	(-0.28)	(-0.74)	(1.22)	
F=9	0.5857	-0.2552***	-0.1288*	-0.0195	-0.1061	0.1014	-0.4070	0.9975	0.7790	-0.5752	0.6986	5.18%
	(1.15)	(-3.22)	(-1.80)	(-0.04)	(-1.05)	(0.54)	(-0.35)	(1.01)	(0.12)	(-1.53)	(0.56)	
F=12	0.6611	-0.3048***	-0.1211*	-0.0577	-0.1069	0.0463	0.7669	0.6661	10.9899	0.2466	-0.8011	5.07%
	(1.36)	(-3.78)	(-1.94)	(-0.17)	(-1.35)	(0.27)	(0.76)	(0.67)	(1.31)	(0.55)	(-0.52)	
						France						
	1.1491***	-0.0583	-0.0205	0.3457**	-0.0958	-0.0074	1.4839**	0.7364	24.5616***	-1.2200***	1.3317***	7.98%

F=1	(3.35)	(-0.98)	(-0.33)	(2.18)	(-1.00)	(-0.06)	(2.08)	(1.59)	(2.69)	(-6.77)	(3.84)	
F=3	1.0209**	-0.0822	-0.1090*	0.1569	0.0098	-0.0335	0.9770	0.3061	21.6134***	-0.3125*	0.3735	6.32%
	(2.65)	(-1.37)	(-1.75)	(1.05)	(0.11)	(-0.28)	(1.19)	(0.69)	(2.75)	(-1.86)	(1.32)	
F=6	0.8513**	-0.0827	0.0360	0.1606	0.0329	-0.0222	1.3050	0.3481	14.9946	-0.2363*	0.4864*	5.75%
	(2.16)	(-1.42)	(0.57)	(0.97)	(0.34)	(-0.18)	(1.64)	(0.80)	(1.61)	(-1.83)	(1.71)	
F=9	0.6775	-0.1133	0.0062	0.2298	0.0269	-0.0163	0.6310	0.5943	-1.0333	-0.0484	0.2009	5.46%
	(1.62)	(-1.61)	(0.10)	(1.39)	(0.30)	(-0.15)	(0.83)	(1.32)	(-0.14)	(-0.41)	(0.72)	
F=12	0.7333*	-0.1394**	0.0415	0.1764	0.0269	-0.0715	0.7503	0.0783	7.3994	-0.1452	0.1313	5.20%
	(1.77)	(-2.11)	(0.58)	(1.24)	(0.29)	(-0.71)	(0.84)	(0.15)	(0.84)	(-1.00)	(0.40)	
						Japan						
F=1	0.3187	-0.0413	-0.1380***	-0.2085*	-0.0434	0.0608	0.1650*	0.1192**	-0.2022	-0.3395***	0.0794*	6.41%
	(1.04)	(-0.74)	(-3.98)	(-1.81)	(-0.49)	(0.60)	(1.87)	(2.21)	(-1.13)	(-6.68)	(1.65)	
F=3	0.4120	-0.0306	-0.0494	-0.1463	-0.0183	0.0271	0.1491	0.1000**	-0.0048	-0.1060**	0.0695	5.94%
	(1.23)	(-0.52)	(-1.47)	(-1.36)	(-0.23)	(0.27)	(1.42)	(2.09)	(-0.03)	(-2.09)	(1.32)	
F=6	0.3507	-0.0497	-0.0380	-	0.0133	0.0377	0.1490	0.0757	-0.0717	-0.1529***	0.0144	5.57%
	(0.95)	(-0.80)	(-1.15)	(-2.03)	(0.16)	(0.36)	(1.61)	(1.35)	(-0.43)	(-2.74)	(0.29)	
F=9	0.3768	-0.0378	-0.0623**	-	0.0240	0.0274	0.1650*	0.0782	-0.0952	-0.0157	-0.0374	5.12%
	(1.09)	(-0.63)	(-2.00)	(-3.52)	(0.32)	(0.29)	(1.85)	(1.19)	(-0.68)	(-0.33)	(-0.83)	
F=12	0.5602	-0.0228	0.0178	-	-0.0303	0.0231	0.1693**	0.1025	-0.1173	-0.0022	-0.0156	4.77%
	(1.58)	(-0.40)	(0.62)	(-4.51)	(-0.35)	(0.25)	(2.04)	(1.55)	(-0.75)	(-0.05)	(-0.31)	

Table 10. International evidence: trading strategy

This table presents the performances of the baseline momentum strategy (WML), the volatility strategy (BSC) proposed by Barroso and Santa-Clara (2015), and the coskewness enhanced volatility strategy (CS-BSC), respectively, for the UK, German, French, and Japanese markets. The target standard deviation and coskewness values are set to the moving average of the monthly standard deviation and coskewness of WML, estimated with data up to month *t-1*, for each of the individual markets. All returns are converted to US dollars and reported in %. See Table 6 for the definition of the statistics. The reporting period is from January 1997 to December 2019.

Holding months	Statistics	UK			Germany			France			Japan		
		WML	BSC	CS-BSC	WML	BSC	CS-BSC	WML	BSC	CS-BSC	WML	BSC	CS-BSC
1	AMEAN	8.17	12.55	10.72	16.19	17.73	15.63	10.59	16.69	12.64	0.70	5.29	3.97
	ASTD	20.00	13.10	10.96	22.84	16.12	12.88	22.44	17.72	12.87	21.22	18.17	15.36
	Sharpe	0.41	0.96	0.98	0.71	1.10	1.21	0.47	0.94	0.98	0.03	0.29	0.26
	Sortino	0.56	1.82	1.95	1.09	2.13	2.71	0.74	1.82	2.02	0.05	0.49	0.42
	SKEW	-3.11	-0.11	0.10	-1.22	-0.21	0.38	-0.64	-0.01	0.29	-1.14	-0.29	-0.29
	MDD	-68%	-47%	-50%	-65%	-42%	-36%	-62%	-42%	-39%	-70%	-51%	-46%
3	AMEAN	8.91	12.90	11.05	14.05	15.63	13.83	9.78	15.09	11.74	-1.11	2.86	2.39
	STD	18.89	12.62	10.72	20.90	15.14	12.19	20.33	15.73	11.41	19.96	16.84	14.10
	Sharpe	0.47	1.02	1.03	0.67	1.03	1.13	0.48	0.96	1.03	-0.06	0.17	0.17
	Sortino	0.66	2.04	2.17	1.06	2.02	2.59	0.74	1.85	2.10	-0.08	0.29	0.28
	SKEW	-2.91	0.08	0.37	-0.89	-0.02	0.71	-0.75	-0.07	0.13	-1.16	-0.15	-0.06
	MDD	-64%	-36%	-37%	-57%	-45%	-36%	-55%	-39%	-36%	-78%	-64%	-58%
6	AMEAN	9.64	13.16	11.46	11.58	13.72	12.05	8.37	12.90	10.20	-1.92	1.78	1.38
	STD	16.98	11.39	9.81	19.70	14.25	11.62	18.64	14.28	10.36	18.11	15.03	12.62
	Sharpe	0.57	1.16	1.17	0.59	0.96	1.04	0.45	0.90	0.98	-0.11	0.12	0.11
	Sortino	0.80	2.41	2.61	0.93	1.96	2.39	0.69	1.72	1.99	-0.15	0.20	0.18
	SKEW	-2.77	0.03	0.36	-0.83	0.24	0.85	-0.84	-0.14	0.04	-1.07	-0.18	-0.10
	MDD	-60%	-35%	-34%	-57%	-45%	-42%	-56%	-40%	-37%	-78%	-58%	-51%
9	AMEAN	9.31	12.47	10.91	9.26	11.73	10.26	6.75	11.02	8.79	-3.23	0.18	-0.03
	STD	15.37	10.60	9.17	18.07	13.26	10.80	17.25	13.20	9.56	16.38	13.76	11.50
	Sharpe	0.61	1.18	1.19	0.51	0.88	0.95	0.39	0.83	0.92	-0.20	0.01	0.00
	Sortino	0.87	2.48	2.66	0.80	1.79	2.12	0.59	1.58	1.86	-0.28	0.02	0.00
	SKEW	-2.37	0.11	0.36	-0.77	0.29	0.82	-0.93	-0.05	0.16	-1.27	-0.33	-0.42

	MDD	-51%	-29%	-30%	-56%	-44%	-42%	-53%	-38%	-35%	-79%	-57%	-58%
12	AMEAN	7.72	10.76	9.46	7.09	9.79	8.53	4.93	8.89	7.15	-4.55	-1.52	-1.47
	STD	14.16	9.77	8.36	16.42	12.46	10.11	15.99	12.38	8.93	14.88	12.73	10.60
	Sharpe	0.54	1.10	1.13	0.43	0.79	0.84	0.31	0.72	0.80	-0.31	-0.12	-0.14
	Sortino	0.77	2.22	2.40	0.67	1.55	1.80	0.46	1.34	1.59	-0.41	-0.18	-0.20
	SKEW	-2.53	0.00	0.21	-0.64	0.29	0.76	-1.06	0.00	0.23	-1.49	-0.52	-0.76
	MDD	-47%	-31%	-27%	-54%	-43%	-41%	-53%	-38%	-36%	-82%	-66%	-70%

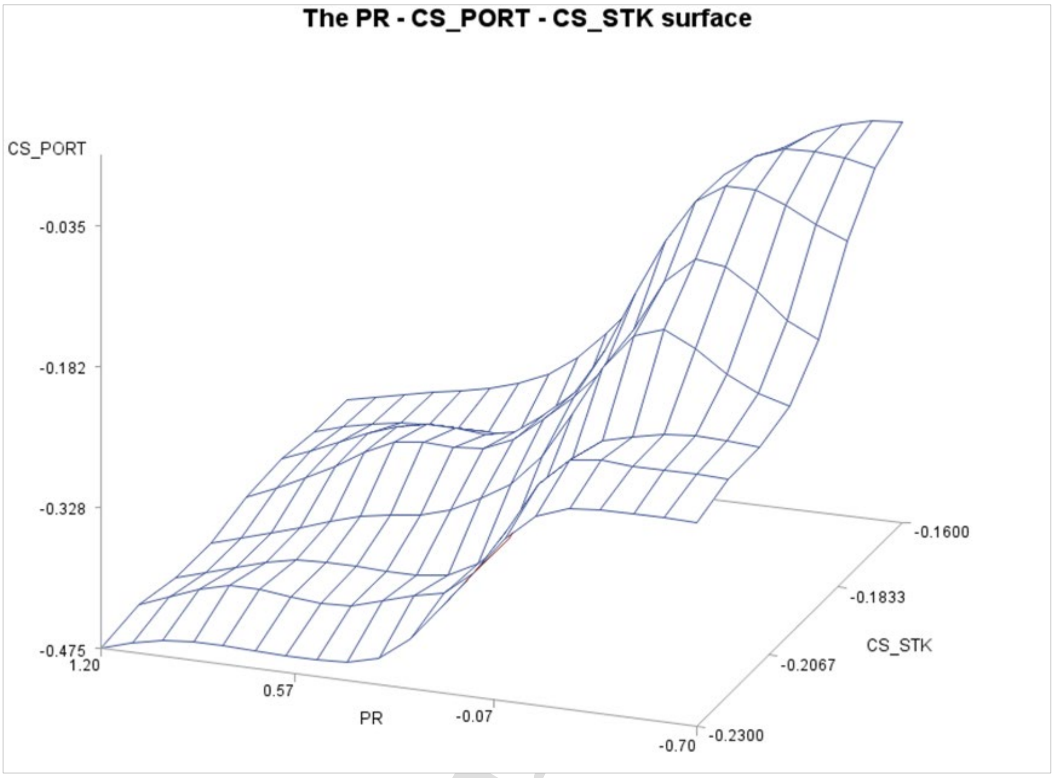


Figure 1. The 3D plot of the PR-CS_STK-CS_PORT surface of the 100 size-PR portfolios. Spline interpolation and smoothing are applied to the surface.

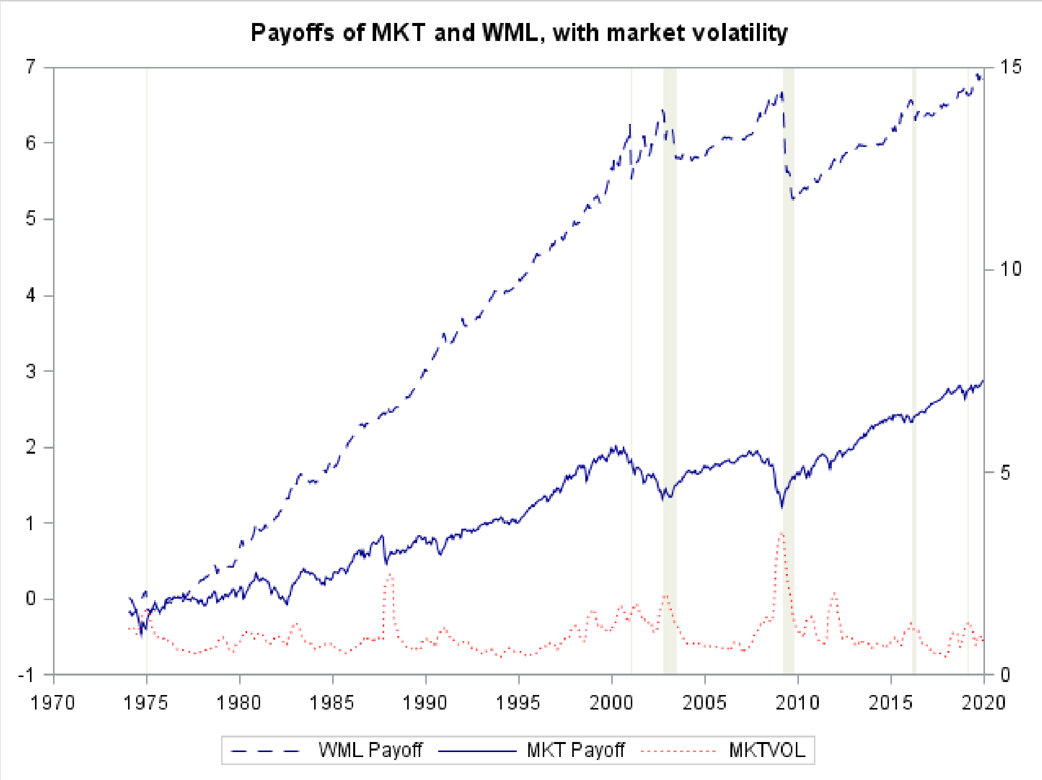
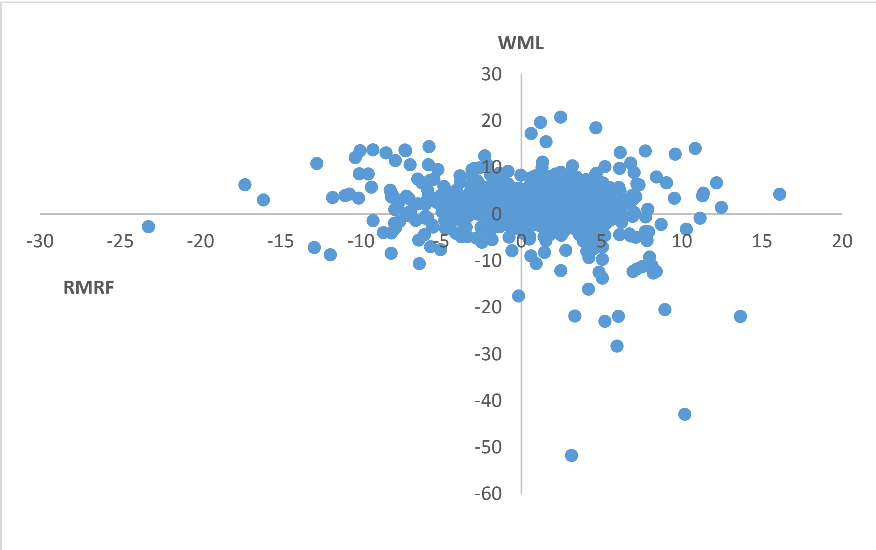
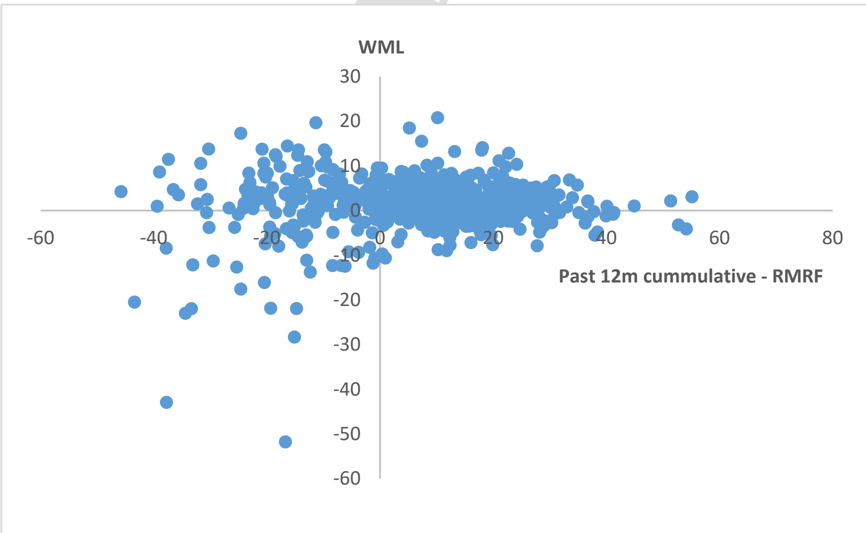


Figure 2. The time series of the cumulative payoff for a $1 investment in the market portfolio and the WML strategy, in comparison with market volatility. The market volatility is measured by the standard deviation of daily market returns over the preceding six months. The cumulative payoff is expressed as powers of ten.

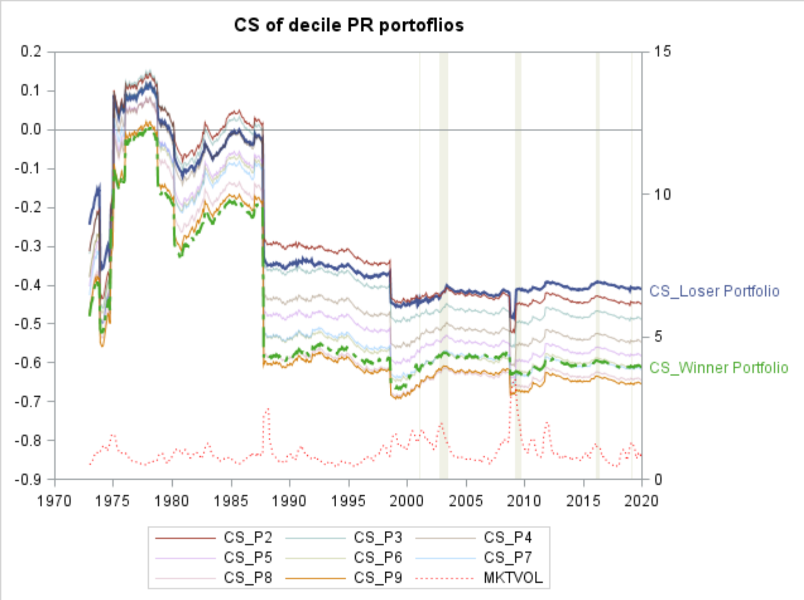


A. Optionality of the WML return. The y-axis is the monthly WML return, and the x-axis is the monthly market return in excess of the risk-free rate.

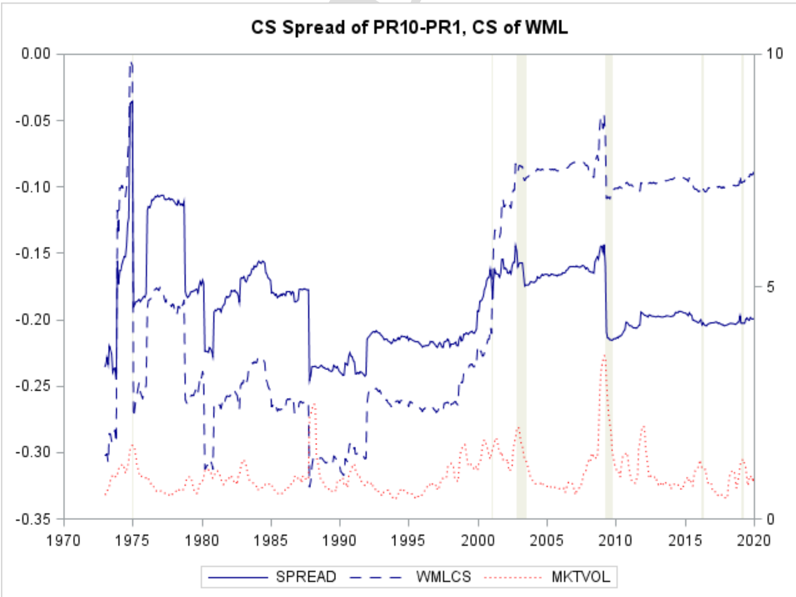


B. Optionality of the WML return. The y-axis is the monthly WML return, and the x-axis is the previous 12-month cumulative excess market return.

Figure 3. The optionality of the WML return.

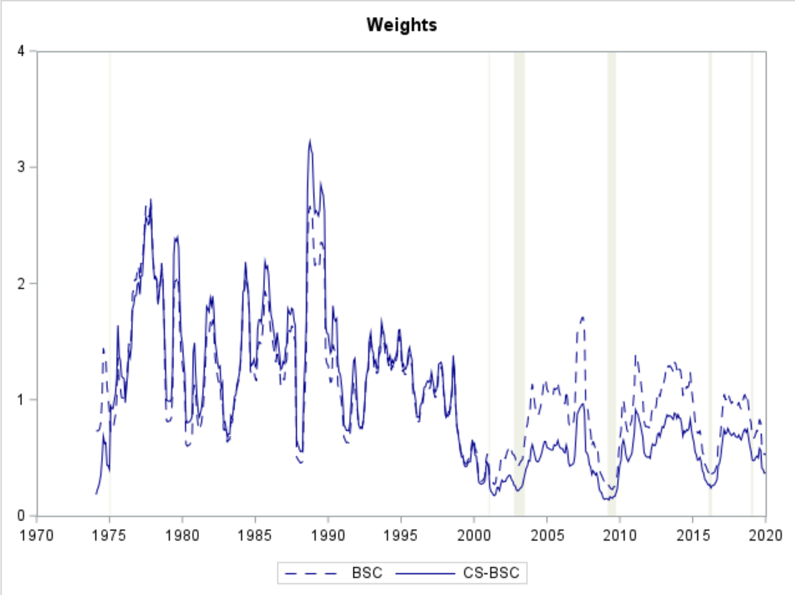


A. The coskewness of the decile momentum portfolios. The shaded areas represent momentum crashes.

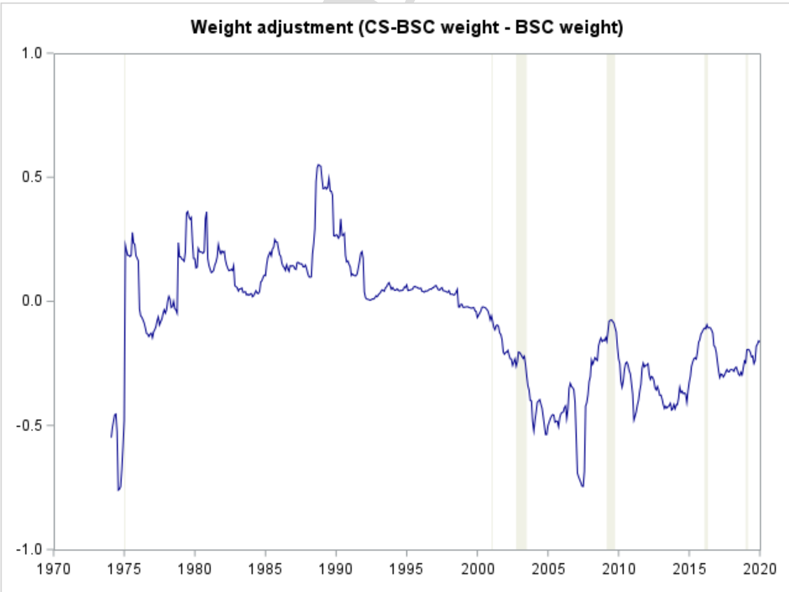


B. The coskewness spread between the winner and loser decile portfolios, and the coskewness of the WML strategy. The shaded areas represent momentum crashes.

Figure 4. Coskewness of momentum portfolios, coskewness spread between the winner and loser decile portfolios, and the coskewness of the WML strategy.

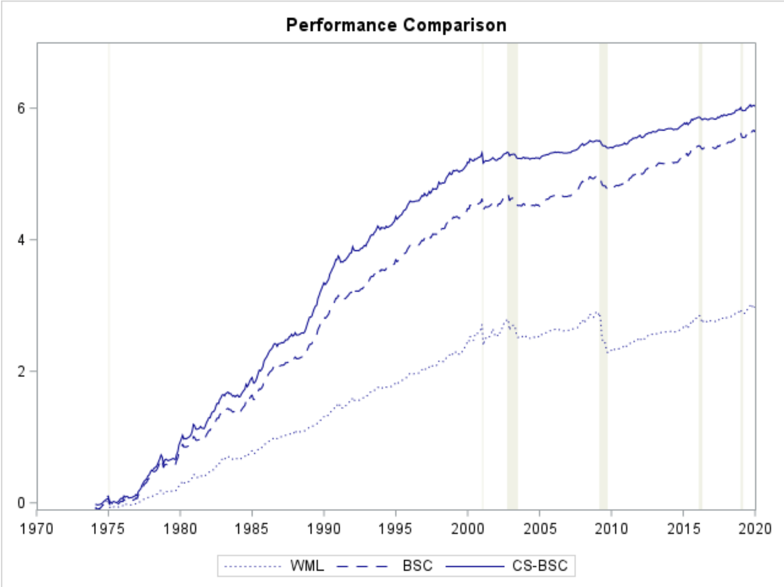


A. The weights of the BSC strategy (dash line) and the CS-BSC strategy (solid line). The shaded areas represent momentum crashes.

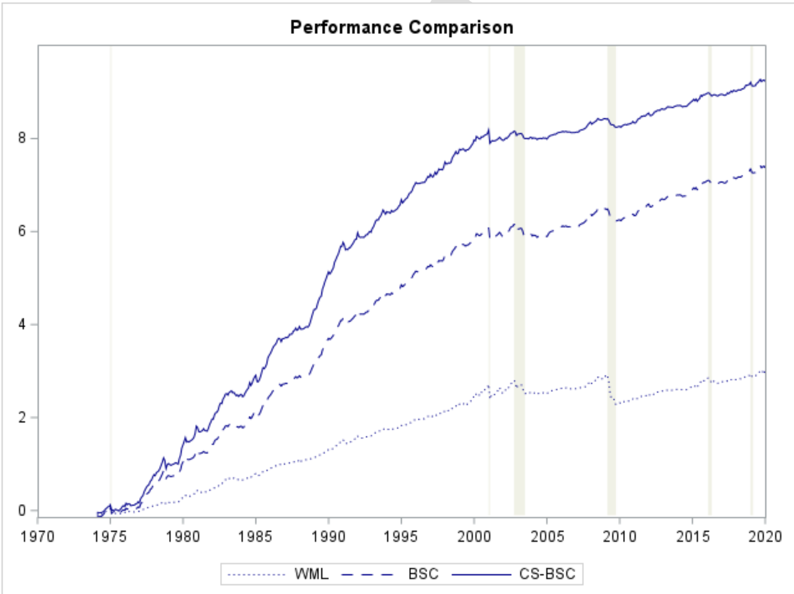


B. The difference in weights between the CS-BSC and BSC strategies. The shaded areas represent momentum crashes.

Figure 5. The weights of the BSC and the CS-BSC strategies and the weight adjustment.



A. The cumulative payoff is expressed as powers of ten. The BSC and CS-BSC strategies are scaled to have the same average volatility as the baseline momentum strategy.



B. The cumulative payoff is expressed as powers of ten. The BSC and CS-BSC strategies are scaled to have the same average downside volatility as the baseline momentum strategy.

Figure 6. The cumulative payoff of a $1 investment at the beginning of 1974 from the baseline momentum strategy (WML), the volatility strategy (BSC) proposed by Barroso and Santa-Clara (2015), and the proposed coskewness enhanced volatility strategy (CS-BSC), respectively. The shaded areas represent momentum crashes.

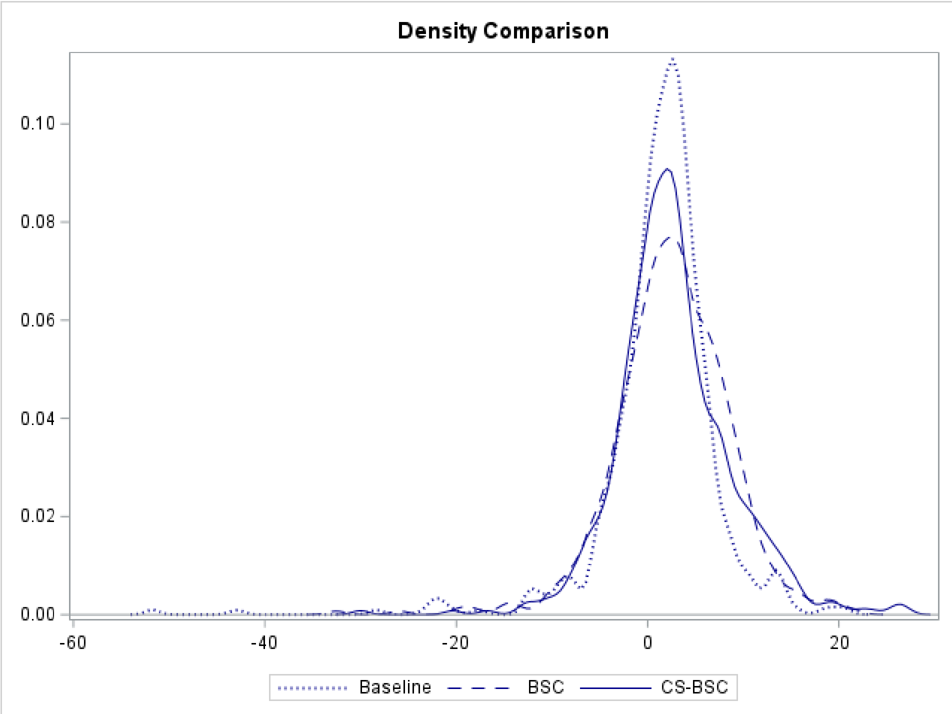


Figure 7. The monthly return density plot of the baseline WML strategy, the volatility BSC strategy, and the CS-BSC strategy, respectively. The BSC and CS-BSC strategies are scaled to have the same average volatility as the baseline WML strategy.

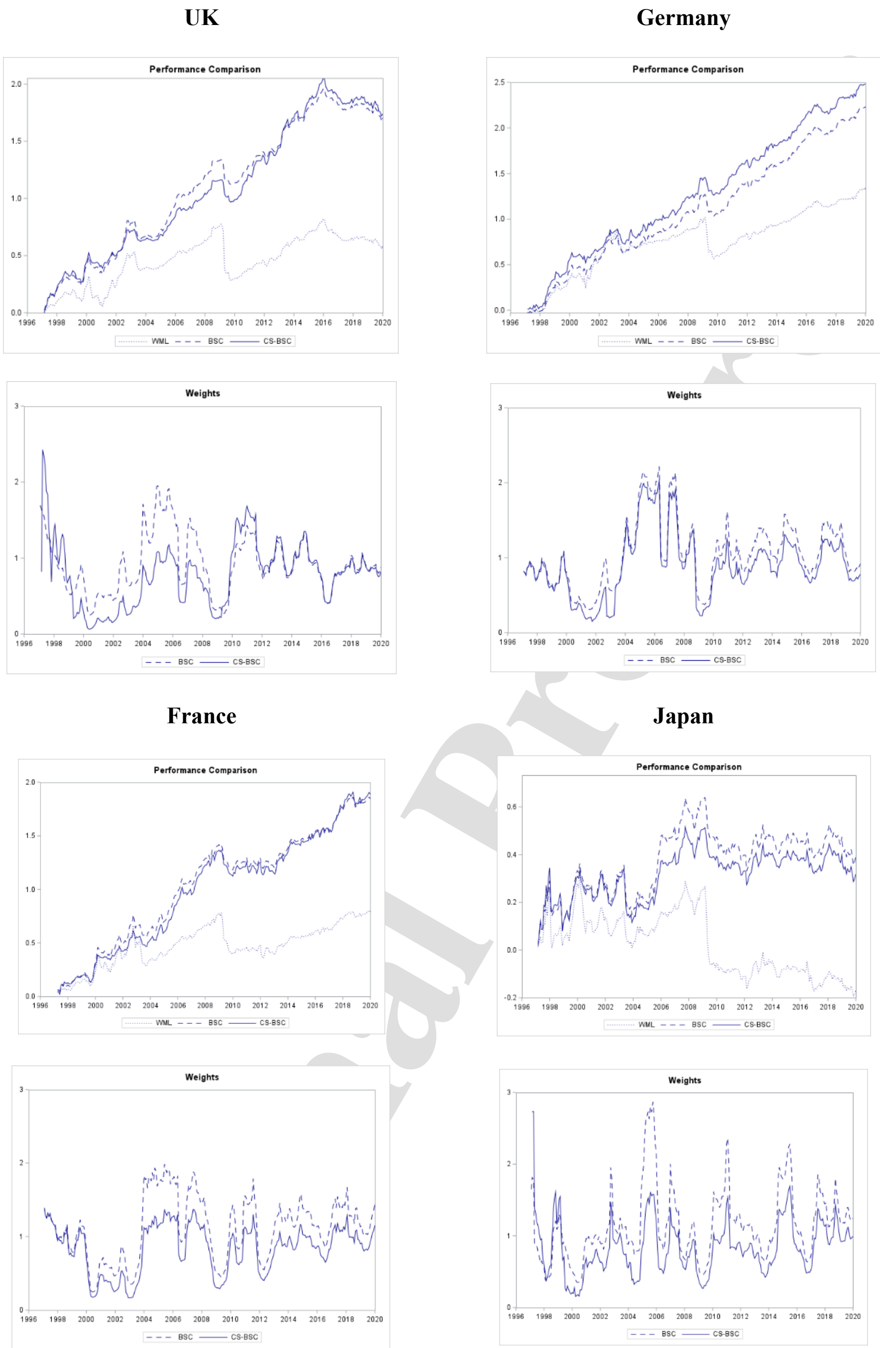


Figure 8. The cumulative payoffs (upper plot) and weights (bottom plot) of the baseline WML, BSC, and CS-BSC strategies for international markets.

Appendix

Table A1: Proposed trading strategy with constant σ^* and CS*

This table presents the performances of the baseline momentum strategy (WML), the volatility strategy (BSC) proposed by Barroso and Santa-Clara (2015), and the coskewness enhanced volatility strategy (CS-BSC), respectively, with constant target volatility and coskewness. The dynamic weight formula is as specified in Table 7. The target σ^* is set to 12% as specified in Barroso and Santa-Clara (2015), CS^* is set to the full-sample mean coskewness at -0.20, and *Range* is the full-sample range of 0.30. See Table 7 for the definition of performance metrics. The reporting period is from January 1974 to December 2019.

Holding	Strategy	AMEAN	ASTD	Sharpe	Sortino	SKEW	MIN	MAX	Maximum
H=1	WML	17.45	20.98	0.83	1.18	-2.58	-51.78	20.76	-76%
	BSC	30.26	20.39	1.48	2.93	-0.50	-32.30	21.53	-36%
	CS-BSC	30.86	19.96	1.55	3.62	0.34	-27.99	25.84	-29%
H=3	WML	15.29	19.51	0.78	1.12	-2.36	-44.51	19.14	-74%
	BSC	27.24	19.23	1.42	2.77	-0.43	-27.77	22.04	-37%
	CS-BSC	28.04	18.90	1.48	3.47	0.47	-24.07	26.11	-25%
H=6	WML	12.40	17.90	0.69	0.99	-2.05	-36.47	16.77	-72%
	BSC	22.77	17.96	1.27	2.44	-0.39	-23.67	20.52	-41%
	CS-BSC	23.58	17.66	1.34	3.00	0.49	-19.72	26.71	-23%
H=9	WML	9.79	16.30	0.60	0.87	-1.73	-29.27	14.22	-69%
	BSC	18.69	16.67	1.12	2.10	-0.34	-22.89	20.95	-42%
	CS-BSC	19.35	16.36	1.18	2.53	0.51	-19.10	27.84	-26%
H=12	WML	7.27	15.11	0.48	0.69	-1.64	-23.10	13.69	-67%
	BSC	14.71	15.66	0.94	1.67	-0.46	-23.97	20.31	-41%
	CS-BSC	15.27	15.23	1.00	2.00	0.41	-21.33	26.99	-29%

Table A2. Strategy performances in subperiods

This table presents the performances of the baseline momentum strategy (WML), the volatility strategy (BSC) proposed by Barroso and Santa-Clara (2015), and the proposed coskewness enhanced volatility strategy (CS-BSC), respectively, in three subperiods. The first subperiod (1974-1990) covers the 1987 crash; the second subperiod (1991-2004) covers the 2001 dot-com crash, and the third subperiod (2004-2019) covers the 2008-2009 GFC. The target σ^* and CS^* are calculated using data from the beginning of the subperiod up to month *t-1*. We report the performances for the 1-month and 6-month holding periods for brevity, and the results for 3-, 9-, and 12-month holding periods also support our conclusions. See Table 7 for the strategy weight functions and the definition of performance metrics.

Holding Period	Strategy	AMEAN	ASTD	Sharpe	Sortino	SKEW	MIN	MAX	Maximum Drawdown
Panel A. 1974-1990 (204 months)									
H=1	WML	21.76	14.20	1.53	2.51	-1.62	-21.98	12.81	-24%
	BSC	32.19	15.41	2.09	4.32	-0.82	-20.66	13.77	-21%
	CS-BSC	37.18	16.79	2.21	5.63	-0.07	-12.55	17.10	-14%
H=6	WML	17.23	13.78	1.25	1.95	-1.79	-22.41	13.55	-28%
	BSC	25.72	14.13	1.82	3.61	-0.86	-21.07	13.46	-25%
	CS-BSC	29.70	15.09	1.97	4.84	0.04	-9.55	18.13	-16%
Panel B. 1991-2004 (168 months)									
H=1	WML	20.57	26.38	0.78	1.13	-2.34	-51.78	20.76	-52%
	BSC	19.78	18.68	1.06	1.75	-1.24	-27.99	18.21	-49%
	CS-BSC	17.94	12.95	1.39	3.32	1.23	-14.60	25.06	-30%
H=6	WML	11.93	21.68	0.55	0.81	-1.68	-36.47	16.77	-53%
	BSC	12.34	16.22	0.76	1.25	-0.80	-19.71	18.85	-53%
	CS-BSC	12.81	11.72	1.09	2.72	2.28	-10.28	25.94	-31%
Panel C. 2005-2019 (180 months)									
H=1	WML	9.65	21.65	0.45	0.59	-2.74	-42.95	15.49	-76%
	BSC	17.01	15.69	1.08	1.96	-0.52	-15.39	14.86	-49%
	CS-BSC	18.75	16.81	1.12	2.11	-0.26	-12.65	15.94	-43%
H=6	WML	7.35	18.12	0.41	0.54	-2.44	-32.99	12.96	-70%
	BSC	13.23	13.25	1.00	1.87	-0.25	-11.82	12.43	-46%
	CS-BSC	14.56	14.57	1.00	1.93	-0.02	-12.23	13.59	-43%

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AUTHORSHIP STATEMENT

Manuscript title: Coskewness and Reversal of Momentum Returns: The US and International Evidence

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Authorship contributions

Please indicate the specific contributions made by each author (list the authors' initials followed by their surnames, e.g., Y.L. Cheung). The name of each author must appear at least once in each of the three categories below.

Category 1

Conception and design of study: T. Y., T. H., _____, _____;

acquisition of data: T. Y., L. Dong, Y. DAI, H. KoT;

analysis and/or interpretation of data: T. Y., L. Dong, Y. DAI, T. H.

Category 2

Drafting the manuscript: T. Y., L. Dong, H. KoT, _____;

revising the manuscript critically for important intellectual content: T. Y., L. Dong, Y. DAI, T. H.

Category 3

Approval of the version of the manuscript to be published (the names of all authors must be listed):

T. Y., L. Dong, Y. DAI, H. KoT, T. H.,

_____, _____, _____, _____, _____.

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This statement is signed by all the authors (*a photocopy of this form may be used if there are more than 10 authors*):

Author's name (typed)	Author's signature	Date
L. Dong	Liang Dong	5.5.2022
Y.K. Dai	Y. DAI	5.5.2022
T. Haque	[illegible]	5.5.2022
H.W. Kot	[illegible]	5.5.2022
T. Yamada	[illegible]	5.5.2022