The Journal of Prediction Markets 2019 Vol 13 No 2 pp 45-62

STUDY ON THE NONLINEAR AND CHAOTIC BEHAVIOR OF EXCHANGE-TRADED FUNDS LISTED IN HONG KONG STOCK EXCHANGES

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ABSTRACT

This study examines the nonlinearity and chaotic behavior of the time series of returns of two exchange-traded funds (ETFs) listed in Hong Kong stock exchanges, namely Hong Kong Tracker Fund (HKTF) and iShares FTSE A50 (ISFT), and the adequacy of autoregressive-generalized autoregressive conditional heteroskedasticity (AR-GARCH) models to capture nonlinearity. A set of nonlinearity tests consistently indicate the presence of nonlinearity in both return time series and the Brock–Dechert–Scheinkman (BDS) test of nonlinearity on AR-GARCH residuals, and the inability of AR-GARCH models to capture the nonlinearity in the return series at different stages of the model-building process. Testing for chaos is a rather delicate part in this study and is done by estimating the correlation dimension for both ETFs' return series. The correlation dimension saturates at a finite value, and the saturation indicates the presence of chaos in two ETFs considered for this study.

Keywords: exchange-traded fund, AR-GARCH, Hong Kong Stock Exchange, nonlinearity, chaotic behavior, time series

INTRODUCTION AND PREVIOUS STUDIES

The first exchange-traded fund (ETF) in the United States was Standard & Poor's 500 Depository Receipts (SPDRs), which was designed to passively mimic the S&P 500 index. Hong Kong Tracker Fund, which is Hong Kong's first ETF, was launched on November 12, 1999. ETFs are passively managed funds intended to closely track the performance of market indices. ETFs combine the benefits of investment diversification through index investing and the flexibility of trading at any time during a market's trading hours. ETFs have become increasingly popular because they represent portfolios of securities designed to track the performance of indices, thereby offering an efficient way for investors to obtain cost-effective exposure. They currently play an increasingly important role in Hong Kong. The number of ETFs has increased to 185 as of December 2017. However, compared to other developed financial markets, the Hong Kong ETF segment is still in a nascent stage.

The aim of this study is to investigate the presence of nonlinear dependence and deterministic chaos in daily returns on two ETFs, namely, Tracker Fund (HKEX stock code: 2800) and X iShares FTSE A50 (HKEX stock code: 2823), by contrasting the random walk hypothesis with chaotic dynamics. The overall monthly turnover of these two ETFs is always more than half of the total monthly turnover of all 185 ETFs. They were listed on November 12, 1999 and November 8, 2004, respectively. Because of the difference in their listing dates, we have to unify the study period from January 1, 2005 to December 31, 2017. If there is presence of nonlinear dependence and deterministic chaos, the predictive ability of the system is strongly limited, especially for long-run predictions. Furthermore, we focus on examining whether chaos theory can explain the complex and random behaviors that may not be explained by stochastic approach or the traditional random walk model. In particular, this study attempts to answer two questions: (1) Does the autoregressive conditional heteroskedastic (ARCH) model capture the presence of nonlinear dependence? (2) Are returns in the sampled Hong Kong ETFs generated by a nonlinear dynamic system?

Whether the financial time series exhibit the evidence of dependence is discussed extensively since Hsieh (1991) cast doubt on the behavior of returns on the S&P 500 Index follow a random walk. Peters (1992, 1994) and Hampton (1996) find the US stock market series are not random; instead, they have long-term dependence. Similar evidence of dependence is also found in other stock markets (Greene and Fieltz, 1977; Papaioannou and Karytinos, 1995; Opong et al., 1999).

Movements in market indices and stock prices have fascinated not only traders, but also academicians and policymakers. Studies on the examination of autocorrelations have found little evidence of linear dependence but latter researches use models developed based on that assumption. Some studies have pointed out that linear models may not be appropriate to capture the complexities of economic and financial time series. In fact, several recent studies have found strong evidence of nonlinearity in the short-term movements of asset returns (Hsieh, 1991, 1993; Lin, 1997; Papaioannou and Karytinos, 1995; Hamill and Opong, 1997; Das and Das, 2006). Identifying nonlinearity will open the possibility of generating nonlinear forecasting models that can better explain certain aspects of stock price returns than do linear forecasting models. Finance and economics are areas that strongly need the application of such an approach because empirical researches show that the linear models are not adequate to explain the underlying dynamics of recent economic data. Some researchers have pointed out that nonlinear dynamics are appropriate to explain the complexity in many stock returns series (Hsieh, 1991, 1993; Brock et al., 1991). Fama (1965) admits that linear modeling techniques are limited in capturing the complicated patterns that chartists claim to see in stock prices.

Chaos has been identified in hydrodynamic turbulence, lasers, electrical circuits, chemical reactions, disease epidemics, biological reactions, and climatic change. The existence of chaotic behaviors can be detected by the following indications: (1) there is an existence of strange attractors characterized by fractal shape; (2) chaotic process is neither random, nor independent and identically distributed (IID); (3) chaotic process is a nonlinear process; (4) chaotic process is a deterministic process, which will retain its dimensionality when it is placed in a higher embedding dimension; and (5) chaotic process is sensitive to initial conditions. Chaotic system is based on the assumptions that the underlying system is a nonlinear deterministic process. Stochastic process models the financial and economic time series as random shocks. Unlike stochastic process, deterministic process models the time series by assuming that there is a long-term memory.

Chaos in financial markets also attracts many researchers, especially when stochastic systems sometimes fail to provide reliable forecasts. Stock price variations generated by a random process with no long-term memory has been prominent in finance research. The random process assumes that stock returns are IID random variables. While the traditional beliefs of random walk and efficient market hypothesis seemed indisputable, Mandelbrot (1964) challenged this traditional assumption by showing that stock price returns did possess a long-term memory. Chaotic behavior can potentially explain fluctuations in economic and financial time series, which appear to be a random process. Other findings suggest that stock returns may be predictable, rather than being a random process (Scheinkman and LeBaron, 1989; Frank and Stengos, 1989; Peters, 1994; Atchison and White, 1996).

As efficient market hypothesis (EMH) and the random walk model seem to fail to capture the behaviors in many stock markets, studies on chaotic behavior and nonlinear dynamics of stock price behavior have been conducted extensively. The published evidence of deterministic chaotic dynamics is controversial. Hsieh (1991) shows that the S&P 500 index returns are governed by low-complexity chaotic dynamics. Scheinkman and LeBaron (1989) report the evidence of deterministic behavior in weekly stock returns. Frank and Stengos (1989) find low-dimensional chaos in gold and silver rates of return. Blank (1991) confirms the evidence of chaos by estimating the correlation dimension and the Lyapunov exponent. Willey (1992) finds that the deterministic characteristic appears in both daily S&P 100 index returns and NASDAQ index returns. Bask (1996, 2002) finds an indication of deterministic chaotic behavior by using the largest Lyapunov exponent in exchange rate series, including Swedish kroner versus Deutche mark, ECU, US dollar, and Japanese yen. Serletis et al. (1997) find evidence of chaotic nonlinearity in east European black-market exchange rate series. Scarlat et al. (2007) find consistent evidence of chaotic dynamics in Romanian financial markets. Das and Das (2007) find that nonlinear structure found in foreign exchange series of 12 countries in their previous studies are deterministic chaos. Iseri et al.

(2008) find evidence of high chaotic phenomena in Istanbul stock exchange. However, Howe et al. (1997) find no evidence of deterministic patterns in Australia and Hong Kong equity returns. Brooks (1998) reports strong evidence of nonlinearity but no evidence of deterministic chaos in the foreign exchange market data. McKenzie (2001) also finds that daily financial markets data for 10 developed markets are not chaotic by using close returns test, although considerable nonlinearities are indicated by the BDS test. Gilmore (2001) finds similar results that daily exchange rate series of British pound, Deutche mark, and Japanese yen are nonlinear dependence and do not support the findings of possible chaos. Serletis and Shintani (2003) find evidence against low-dimensional chaos in Dow Jones Industrial Average and suggest using stochastic models and statistical inference in the modeling of asset markets. De Grauwe and Vansteenkiste (2007) mostly find no indication of chaotic dynamics in the foreign exchange markets. Mishra et al. (2011) find little evidence to support six Indian stock market index series that are generated by a chaotic system, although the presence of serial correlation, marginal persistence, and nonlinear dependence are indicated. Madhavan (2013a) reveals the prevalence of nonchaotic nonlinearity in two chosen US and European credit default swap (CDS) indices, namely CDX.NA.IG and iTraxx.Europe. Madhavan (2013b) also finds the absence of chaotic behaviors in any of the Indian-listed ETFs considered in the study and shows that all the nonlinearities could not be captured by appropriate GARCH models, except one sampled ETF. BenSaida and Litimi (2013) conclude that previous studies are inconclusive about the presence of chaotic behavior in a financial series because of test misspecification, as the chaos test were designed for clean data. It is generally difficult to have a conclusive evidence of chaotic dynamics in financial and economic time series because chaos analysis techniques cannot separate exogenous noise from chaos. Owing to such controversial results, there is a need for providing more evidence of chaotic behavior in financial time series.

The remainder of this paper is organized as follows: Section 2 describes the data used in this study. Section 3 illustrates the methods employed in this study, while Section 4 presents the empirical results. Section 5 provides the general conclusions and implications for the financial analysts.

DATA

Our sample contains the daily closing prices of Hong Kong Tracker Fund (HKTF) and iShares FTSE A50 (ISFT), the two major ETFs listed on the Hong Kong Exchanges, during the period 2005–2017. The daily prices of the sampled ETFs were obtained from Datastream (Thomson Financial Limited) and were checked against the prices supplied directly by investment managers. HKTF and ISFT were issued on November 12, 1999 and November 15, 2004, respectively. Therefore, our sample period starts on January 1, 2005. Another reason for this choice of sample period is the dramatic increase in the volatility of stock markets during 2007–2008. Because of the synchronization, the analyzed periods are shorter than the actual periods since the introduction of these two ETFs.

The logarithmic returns of the sampled ETFs may be computed as follows:

$$R_t = \ln\left(\frac{X_t}{X_{t-1}}\right), \tag{1}$$

where X_t is the closing ETF price on date t, and R_t is the daily logarithmic rate of return. Taking first logarithmic difference not only may ensure that our time series are stationary and "whitened" but also is an effective way to compute the continuously compounded rates of returns, although Chen (1988) argues that taking first difference may destroy any delicate nonlinear structure. Peters (1989, 1992) points out that taking first logarithmic difference may filter out the economic growth effect inherent in the stock market series.

The time series of closing prices and logarithmic returns of the sampled ETFs are then subjected to the Augmented Dickey–Fuller (ADF) test to evaluate their order of integration. The ADF statistics of the unit root test indicate that the time series of daily closing prices of HKTF and ISFT are I(1) in levels and those of logarithmic returns are I(0). Table 1 presents the descriptive statistics of the closing price series and logarithmic return series.

Table 1. Summary Statistics

	Tracker Funds (TFHK)	**iShares FTSE A50 (ISFT)**	**Log. Returns of Tracker Funds**	**Log. Returns of iShares FTSE A50**
Mean	21.43	11.26	0.00009	0.00016
Median	22.00	11.04	0.00000	0.00000
Maximum	32.15	27.70	0.04680	0.07782
Minimum	11.60	3.53	–0.04948	–0.07152
Standard Deviation	3.84	4.15	0.00626	0.00883
Skewness	–0.3021	0.9517	–0.1647	0.07075
Kurtosis	2.8291	5.3200	10.2724	11.8693
Jarque-Bera (JB) Statistics	55.6866*	1272.0580*	7485.8270*	11114.4212*
ADF Statistics [I(0)]	–2.0768	–1.9593	–60.5478*	–44.0400*
ADF Statistics [I(1)]	–60.6276*	–46.0123*		

Note: *Significance at 1 percent level.

RESEARCH METHODS

A brief description of a set of methods to detect the nonlinearity and chaotic behavior in the time series are introduced in this section. Not all nonlinearity tests may detect all types of nonlinearity, as their process may have different underlying data-generating mechanism. Currently, there is no single reliable statistical test for the presence of chaotic behavior in time series. A feasible way to detect nonlinear and chaotic structure in time series is to adopt all available tests, in order to avoid misleading results and conclusions (Papaioannou and Karytinos, 1995; Gilmore, 1996).

TESTING FOR NONLINEARITY: MCLEOD-LI TEST

The McLeod-Li test was originally proposed to test for ARCH effect based on the Ljung–Box test (McLeod and Li, 1983). It tests whether the first *L* autocorrelations for the squared residuals of the prewhitened data are collectively small in magnitude. The lag *k* squared sample autocorrelation of squared residuals (ε_t^2) is estimated as follows:

$$\hat{r}_k^2 = \frac{\sum_{t=k+1}^{n} \varepsilon_t^2 \varepsilon_{t-k}^2}{\sum_{t=1}^{n} \varepsilon_t^2}. \tag{2}$$

The Ljung–Box *Q*-statistic of McLeod-Li test for the first *L* autocorrelations is given by

$$Q_{STAT} = n(n+2)\sum_{k=1}^{L} \frac{\hat{r}_k^2(\varepsilon_t^2)}{n-k}. \tag{3}$$

Under the null hypothesis that the error term derived from an autoregressive integrated moving average (ARIMA) model is being generated by an IID process, the test statistic has a χ^2-distribution with a degree of freedom of *L*. Not rejecting the null hypothesis implies that the data is generated by a linear mechanism.

TEST FOR NONLINEARITY: ENGLE LM TEST

Engle's ARCH test is a Lagrange multiplier (LM) test to assess the significance of ARCH effects and may be used to test nonlinearity in second moments (Engle, 1982). The null hypothesis of the Engle LM test is $H_0 : \alpha_0 = \alpha_1 = \cdots = \alpha_m = 0$ in the following auxiliary regression:

$$e_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \cdots + \alpha_m e_{t-m}^2 + u_t. \tag{4}$$

The alternative hypothesis is then the existence of autocorrelation in the squared residuals. The test statistic for the Engle LM test is the conventional *F* statistic of the auxiliary regression presented in equation (4). Under the null hypothesis of linear data mechanism, the *F* statistics follows a χ^2 distribution with *m* degree of freedom. The value of *m* may be determined by comparing the log-likelihood values for different *m*.

TEST FOR NONLINEARITY: HINICH BISPECTRUM TEST

The basis of the Hinich bispectrum test is that linear data-generating processes have no nonzero bicorrelations (Hinich, 1982). Assuming that time series $\{X_t\}$ is a third-order stationary stochastic process and has zero mean for all *t*, the bispectrum is a double Fourier transformation of its third-order bicovariance function at the frequency pair (f_1, f_2) and given by

$$B_X(f_1,f_2)=\sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty}E[X_t,X_{t-m},X_{t-n}]e^{-2\pi i(f_1 m+f_2 n)}. \tag{5}$$

The bispectrum implies that there exists a triangular principal domain

$$\text{Dom} = \{0 < f_1 < 0.5, f_2 < f_1, 2f_1 + f_2 < 1\}.$$

The consistent estimator of the bispectrum is obtained by smoothing the third-order periodogram $F_X(j, k)$ over adjacent frequency pairs as

$$\hat{B}_X(f_m,f_n)=\frac{1}{M^2}\sum_{j=(m-1)M}^{mM-1}\sum_{k=(n-1)M}^{nM-1}F_X(j,k), \tag{6}$$

where $F_X(j, k) = X(f_j)\, X(f_k)\, X^*(f_{j+k})$ and $X(f_j)=\sum_{t=0}^{N-1}X_t e^{-i2\pi f_j t}$ are the Fourier transformation of the sample time series.

The null hypothesis of Gaussianity and linearity can be tested using the estimated standardized bispectrum, which is asymptotically a chi-square distribution with two degrees of freedom as

$$\hat{X}(f_m,f_n)=\frac{\hat{B}_X(f_m,f_n)}{\sqrt{\frac{NQ_{m,n}}{M^4}\left[\hat{S}_X(f_m)\hat{S}_X(f_n)\hat{S}_X(f_{m+n})\right]}}. \tag{7}$$

If the standardized bispectrum in the frequency domain is not flat as a function of frequency pairs, a nonlinear data generating process is implied.

TEST FOR NONLINEARITY: HINICH BICORRELATION TEST

The Hinich bicorrelation test requires that the serial linear dependence should be removed from the time series before performing the test. The removal may be achieved by fitting an autoregressive model, and it will ensure that the remaining serial dependence is attributed to nonlinear generating process. The null hypothesis is that time series $\{X_t\}$ is a stationary white noise process that has a zero bicorrelation. The alternative hypothesis is that the data-generating process has a nonzero bicorrelation. The Hinich bicorrelation test statistic has a chi-square distribution with $L(L-1)/2$ degrees of freedom as

$$H = \sum_{s=2}^{L} \sum_{r=1}^{s-1} G^2(r,s), \tag{8}$$

where $G^2(r,s) = (n-s)^{1/2} C_3(r,s)$ and $C_3(r,s) = \frac{1}{n-s} \sum_{t=1}^{n-s} e_t e_{t+r} e_{t+r+s}$ for $0 \le r \le s$.

TEST FOR CHAOS: BDS TEST

Brock et al. (1987) suggested the BDS test, which is a powerful tool for detecting serial dependence in time series based on the concept of correlation integrals. The BDS test tests the null hypothesis of IID against an unspecified alternative using a nonparametric approach. If the linear dependence has been removed in the time series, rejecting the null hypothesis indicates that the serial dependence is nonlinear.

However, the BDS test cannot distinguish between nonlinear deterministic chaos and nonlinear stochastic systems. While it cannot test chaos directly, it can test only nonlinearity, provided that any linear dependence has been removed from the data (e.g., by using traditional GARCH-type filters or taking a first difference of natural logarithms). Nevertheless, as *nonlinear* dependence is one of the indications of chaos, we may use the BDS test to detect such an indication. The BDS test statistic is given by

$$BDS_{\varepsilon,m} = \frac{\sqrt{n}[C_{\varepsilon,m} - (C_{\varepsilon,1})^m]}{\sqrt{V_{\varepsilon,m}}}, \tag{9}$$

where $C_{\varepsilon,m} = \left[\frac{1}{n_m(n_m-1)} \sum_{i \neq j} I_{i,j;\varepsilon} \right]$ is the correlation integral that measures the spatial correlation among the points and can be computed by adding the number of pairs of points (*i*, *j*), which are within a radius or tolerance ε of each other; $I_{i,j;\varepsilon}$ is the indicator function given by $I_{i,j;\varepsilon} = \begin{cases} 1 & if \quad \left\| x_i^m - x_j^m \right\| \le \varepsilon \\ 0 & \text{otherwise} \end{cases}$.

$\sqrt{V_{\varepsilon,m}}$ is an estimate of the asymptotic standard deviation of $C_{\varepsilon,m} - (C_{\varepsilon,1})^m$. Brock et al. (1987) show that if time series $\{X_t\}$ is IID, then the correlation integral at embedding dimension (m) and given a certain value of ε, can be approximated by the m^{th} power of $C_{\varepsilon,1}$, in which the correlation integral under the embedding dimension equals 1. In other words, a significant positive BDS statistics indicates that certain patterns are too frequent rather than a true random process. The test is generally conducted on the residuals of a linear or GARCH-type filter. This test will be repeated at different values of *e* and *m*. Lin (1997) suggests that the appropriate value of ε/σ ranges from 0.5 to 2 and Brock et al. (1987) propose that the appropriate value of *m* should be between 2 and 5. A standardized BDS test statistic value that is greater than +1.96 would imply a rejection of null hypothesis.

TEST FOR CHAOS: CORRELATION DIMENSION

The correlation dimension estimation may be used to differentiate deterministic chaos from stochastic systems. Peters (1991a) states that a fractal shape retains its dimensionality when it is placed in a greater embedding dimension. The increase in fractal dimension of a chaotic process will not synchronize the increase in embedding dimension. Liu, Granger, and Heller (1993) prove that for a true chaotic system, the correlation dimension has a stabilized value when the embedding dimension increases.

Peters (1991b) indicates that the fractal dimension may be calculated by using correlation dimension that measures how an attractor fills its space. Given an embedded time series $\{X_i^m\} = [x_1^m, x_2^m, x_3^m, \ldots\ldots x_{N-m}^m]$, correlation integrals $C_{\varepsilon,m}$ may be calculated according to the algorithm illustrated in equation (9) for different values of ε.

There are several ways of estimating correlation dimension v_m. The most common practice in estimating v_m is by the OLS regression model where the dependent variable is the natural logarithm of correlation integral $[ln(C_{\varepsilon,m})]$ and the independent variable is the natural logarithm of tolerance $[ln(\varepsilon)]$ (Denker and Keller, 1986; Scheinkman and LeBaron, 1989). The regression coefficient will then be the estimated correlation dimension. This regression equation is as follows:

$$ln(C_{\varepsilon,m}) = v_m \, ln(\varepsilon) + c. \tag{10}$$

We repeat the estimations of v_m with different values of *m*. A plot of embedding dimension (*m*) along the *x*-axis against their corresponding estimated correlation dimension (v_m) along the y-axis is then constructed. If chaotic behavior is present in the time series, the estimated correlation dimension (v_m) will stabilize at a certain value when embedding dimension (m) continues to increase. If this stabilization does not occur, it implies that v_m increases without bound as m increases, then the system is stochastic rather than chaotic.

RESULTS

NONLINEARITY TEST

The time series of logarithmic returns of sampled ETFs, HKTF, and ISFT are first subjected to autoregressive (AR) models. This may remove the linear dependence from the time series. Any dependence left in the AR-filtered residuals diagnosed by any test may be regarded as nonlinear dependence. The optimum lags for such AR filters may be determined by the Akaike information criterion (AIC). The optimum lags are found to be 2 and 7 for HKTF and ISFT, respectively.

After the logarithmic return time series have been filtered by the AR filters, the AR residuals of each time series will then be subjected to different tests of nonlinearity. The set of nonlinearity tests employed in this study are McLeod-Li test, Engle LM test, Hinich bispectrum test, and Bicorrelation (H) test. The details of these tests are introduced in the previous section. The test statistics, summarized in Table 2, are all significant at the 1% level and indicate the presence of nonlinear dependence in the time series.

TESTING THE ADEQUACY OF AR-GARCH MODELS

The multiple nonlinearity tests show that the time series are nonlinearly dependent. The time series of the sampled ETFs will then be subjected to GARCH model to investigate whether such nonlinear dependence may be captured by it. As the optimum lag of AR model is found to be 2 and 7 for HKTF and ISFT, respectively, AR(2)-GARCH(p, q) and AR(7)-GARCH(p, q)

Table 2. Nonlinearity Test Results

	Tracker Funds (TFHK)	**iShares FTSE A50 (ISFT)**
McLeod-Li Test Statistics	30694.237*	32082.301*
Engle LM Test Statistics	47319.894*	73855.659*
Hinich Bispectrum Test Statistics	322.089*	284.026*
Bicoreelation H-Statistics	529544.695*	828952.121*

Note: *Significance at 1 percent level.

will then be employed to capture the dependence in the respective time series. The appropriateness of the selected AR-GARCH will be examined by LM test to the AR-GARCH residuals.

In the case of HKTF, the residuals are diagnosed as having the ARCH effect if AR(2)-GARCH(1,1) is used. The residuals are found as not having the ARCH effect until AR(2)-GARCH(5,1) model is fitted. Similarly, in the case of ISFT, no residual ARCH effects are found until AR(7)-GARCH(3,1) model. Table 3 presents the variance equation estimates and the test results of residual diagnosis of the AR-GARCH models fitted to the residuals.

The BDS test is then employed to test the adequacy of the fitted AR and AR-GARCH models to capture the nonlinear structure in the time series. The BDS test statistics are calculated for different values of embedding dimension ranging from 2 to 5, and for different threshold values, which are the ratios of tolerance to standard deviation ε/σ ranging from 0.5 to 2.0. Although the test results are impacted by the author's choice of embedding dimension and threshold value, the selected ranges of values are commonly used figures. Table 4 presents the BDS test results.

The results indicate that all the BDS test statistics are significant at the 1% level under different combinations of embedding dimension and threshold value ε/σ. Thus, we should reject the null hypothesis of IID for the residual series obtained from fitting AR and AR-GARCH models on both return series of HKTF and ISFT. Regarding the case of fitting AR models, the BDS test statistics indicate the existence of nonlinear dependence in the time series and the results are consistent with the set of nonlinearity test results. The BDS test is then used to test the existence of nonlinear dependence in the AR-GARCH residual series. Rejecting the null hypothesis of IID for the residual series

Table 3. Variance Equation Estimates of AR-GARCH Models

	Tracker Funds (TFHK)	**iShares FTSE A50 (ISFT)**
Variance Equation Estimates		
α	0.0002**	0.0001**
β_1	0.3288*	0.8167*
β_2	–0.2912*	–0.8031*
β_3	–0.0061	0.0405
β_4	0.0379	
β_5	–0.0260	
ω_1	0.9544*	0.9478*
Residual Diagnosis		
F-statistic	1.7363	2.4503
nR^2	8.6747	7.3437

Note: **Significance at 5 percent level; *Significance at 1 percent level.

Table 4. BDS Test Results

		Tracker Funds (TFHK)		iShares A50 (ISFT)	
ε / σ	Embedding Dimension (*m*)	AR(2) Residuals	AR(2) – GARCH(5,1) Residuals	AR(7) Residuals	AR(7) – GARCH(3,1) Residuals
2	2	137.92*	58.30*	107.93*	56.46*
2	3	136.69*	52.19*	104.50*	54.88*
2	4	135.95*	46.81*	101.39*	52.86*
2	5	137.64*	42.67*	100.04*	51.23*
1.5	2	139.71*	55.73*	116.41*	61.63*
1.5	3	148.89*	49.88*	120.14*	61.45*
1.5	4	160.16*	44.75*	124.78*	61.05*
1.5	5	176.43*	40.79*	132.35*	61.21*
1	2	150.39*	28.56*	128.90*	61.63*
1	3	182.80*	26.12*	152.48*	61.45*
1	4	227.62*	23.69*	184.15*	61.05*
1	5	294.35*	22.14*	230.26*	61.21*
0.5	2	212.29*	27.58*	219.50*	90.49*
0.5	3	359.59*	30.84*	360.97*	113.83*
0.5	4	656.51*	31.45*	637.17*	150.25*
0.5	5	1293.26*	31.45*	1209.87*	210.16*

Note: The test statistics shown in the table are standardized Z-statistics.

*Significance at 1 percent level.

obtained from AR-GARCH models indicates that the selected AR-GARCH models may not capture the nonlinear dependence in the series.

TESTING FOR CHAOS

To test the presence of chaos, we estimated the correlation dimensions of both return series under different embedding dimensions (*m*) = 1, 2, …., 20, with the delay time (τ) = 1. Table 5 presents the estimated correlation dimensions (v_m) of both return series under different embedding dimension (*m*). Figure 1 presents the plots of the estimated correlation dimensions (v_m) against their respective underlying embedding dimension (*m*) of both return series and a series of random data for comparison.

Three distinct regimes may be observed in Figures 1(a) and 1(b). When the embedding dimension is less than 5, the value of correlation dimension increases rapidly as the value of embedding dimension increases. However, when the value of embedding dimension is still low, it is very hard to distinguish

Table 5. Estimated Correlation Dimension

Embedding Dimension (*m*)	AR(2) – GARCH(5,1) Residuals of Tracker Fund (TFHK)	AR(7) – GARCH(3,1) Residuals of iShares A50 (ISFT)	Random
1	1.68	2.16	0.94
2	2.03	2.38	1.86
3	2.14	2.46	2.80
4	2.19	2.50	3.91
5	2.22	2.53	4.95
6	2.24	2.55	5.83
7	2.24	2.56	6.92
8	2.26	2.58	7.91
9	2.27	2.59	8.90
10	2.28	2.59	9.92
11	2.28	2.60	10.90
12	2.28	2.61	11.88
13	2.28	2.61	12.91
14	2.28	2.62	13.90
15	2.28	2.62	14.92
16	2.28	2.63	15.86
17	2.29	2.63	16.88
18	2.28	2.63	17.90
19	2.28	2.63	18.89
20	2.28	2.64	19.91

that the underlying system is a chaotic or a stochastic system. The indication of chaos is masked by the noise. When the embedding dimension increases ($5 < m < 10$), there is a weak tendency to saturate at a constant value. This behavior indicates that our system may be deterministic. When the embedding dimension is relatively large ($10 < m < 20$), the value of correlation dimension does not follow the increases in embedding dimension. The slope of the straight line becomes very small and saturates. This saturation of the estimated correlation dimensions indicates the underlying system of both ETFs as a chaotic process rather than a random process. The saturated correlation dimensions presented in Table 5 are all nonintegers; the strange attractor emerges in our series and we may conclude that our series are not only deterministic but also chaotic due to the existence of chaotic attractor.

Figure 1 shows a graph of estimated correlation dimension against embedding dimension for the simulated random data; the correlation dimension follows the increase in embedding dimension. It is found that the correlation dimension is infinite for a random series because it is disordered and fills the whole phase space.

Figure 1 (a)
AR(2) - GARCH (5,1) Residuals
Tracker Fund (TFHK)

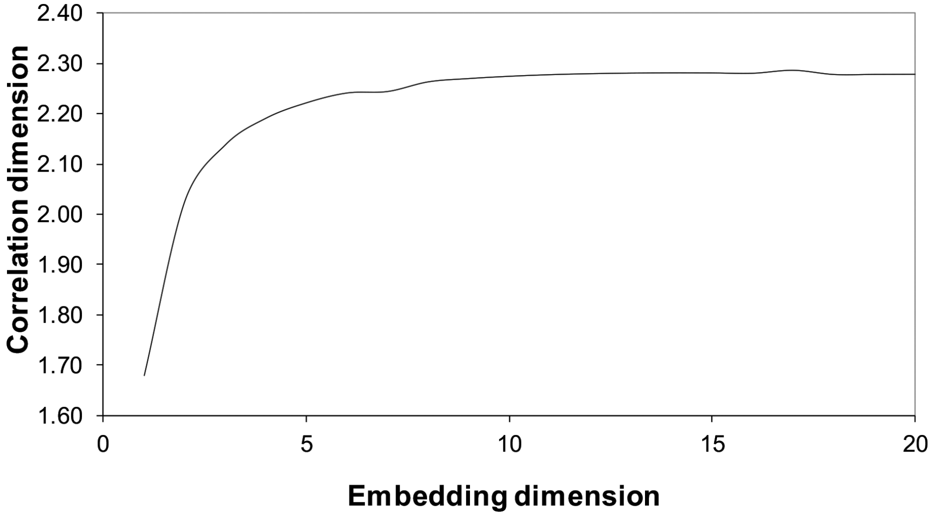
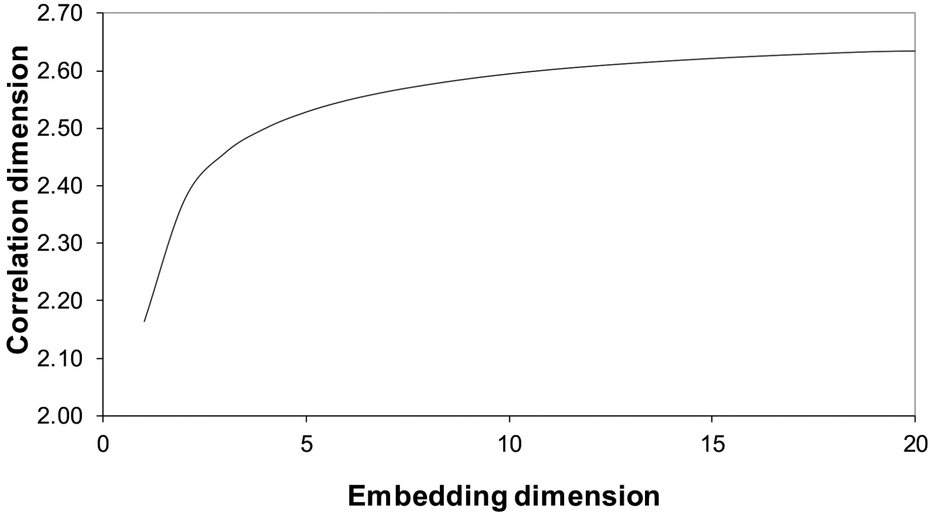


Figure 1 (b)
AR(7) - GARCH (3,1) Residuals
iShares A50 (ISFT)



CONCLUSION

In this study, we have examined the nonlinearity and chaotic behavior of the time series of returns of two exchange-traded funds (ETFs) listed in Hong Kong stock exchanges (HKEX): Hong Kong Tracker Fund (HKTF) and iShares FTSE A50 (ISFT). Their trading volumes are more than half of the total trading volumes of all ETFs listed in HKEX. The adequacy of AR-GARCH models to capture nonlinearity is also examined in this study.

As no single nonlinearity test is superior to another, a set of nonlinearity tests including McLeod-Li test, Engle LM test, Hinich bispectrum test, and Bicorrelation (H) test are employed and all of them indicated the presence of nonlinearity in both return time series.

Both return time series are then modeled by appropriate AR-GARCH models. The BDS test of nonlinearity on AR-GARCH residuals indicates that the nonlinearity in return series cannot be captured by AR-GARCH models at different stages of the model-building process. The results are consistent under different choices of embedding dimension and different threshold values. Nonlinearity is one of the indications of chaotic behavior.

Testing for chaos is a rather delicate part is this study. Correlation dimension is estimated for both ETFs' return series at different embedding dimensions. The plots of the values of correlation dimension against the respective values of embedding dimension show that the correlation dimension saturates at a finite value after the embedding dimension is greater than six. The saturation indicates the presence of chaos in both ETFs considered for this study.

To conclude, the findings of nonlinear process and chaotic behavior in the return time series of the two major ETFs in HKEX provide some insights to financial analysts and economists. First, we may not be able to apply traditional econometric tools when modeling the Hong Kong-listed ETF returns because most of them try to whiten the originally nonlinear data to suit the linear-based tools. Second, the underlying nonlinear process makes the back testing of models, and performances have little meaning for the future. Third, the chaotic behavior found indicates that the price movements of Hong Kong-listed ETF may not be easily modeled by the traditional econometric tools, especially in the long term due to the sensitivity to initial condition, which is one of the characteristics of chaos. However, prediction is possible in the short term, as the chaotic process is a deterministic system. Fourth, the findings of nonlinearity in AR-GARCH filtered residuals show the possibility of the prevalence of additive nonlinearity in conjunction with multiplicative nonlinearity.

REFERENCES

Atchison, MD, and MA White. 1996. "Disappearing Evidence of Chaos in Security Returns: A Simulation." *Quarterly Journal of Business and Economics* 35: 21–37.

Bask, M. 1996. "Dimensions, and Lyapunov Exponents from Exchange Rate Series." *Chaos Solitons & Fractals* 7(12): 2199–2214.

Bask, M. 2002. "A Positive Lyapunov Exponent in Swedish Exchange Rates? *Chaos Solitons & Fractal* 14(8): 1295–1304.

BenSaida, A, and H Litimi. 2013. "High Level Chaos in the Exchange and Index Markets." *Chaos, Solitons & Fractals* 54: 90–95.

Blank, S. 1991. "'Chaos' in Futures Markets? A Non-Linear Dynamical Analysis." *Journal of Future Markets* 11(6): 711–28.

Brock, WA, WD Dechert, and JA Scheinkman. 1987. "A Test for Independence Based on the Correlation Dimension." Working Paper No. 8702, Department of Economics. University of Wisconsin at Madison.

Brock, WA, DA Hsieh, and B LeBaron. 1991. *Nonlinear Dynamics, Chaos, and Instability: Statistical Theory and Economic Evidence.* MIT Press.

Brooks, C. 1998. "Chaos in Foreign Exchange Markets: A Skeptical View." *Computational Economics* 11(3): 265–81.

Chen, P. 1988. "Empirical and Theoretical Evidence of Economic Chaos." *System Dynamics Review* 4(1–2): 81–108.

Das, A, and P Das. 2006. "Does Composite Index of NYSE Represents Chaos in the Long Time Scale?" *Applied Mathematics and Computation* 174(1): 483–89.

Das, A, and P Das. 2007. "Chaotic Analysis of the Foreign Exchange Rates." *Applied Mathematics and Computation* 185(1): 388–96.

De Grauwe, P, and I Vansteenkiste. 2007. "Exchange Rates and Fundamentals: A Non-Linear Relationship?" *International Journal of Finance and Economics* 12(1): 37–54.

Denker, G, and G Keller. 1986. "Rigorous Statistical Procedures for Data from Dynamical Systems." *Journal of Statistical Physics* 44(1–2): 67–93.

Engle, RF. 1982. "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation." *Econometrica* 50: 987–1007.

Fama, EF. 1965. "The Behavior of Stock Market Prices." *Journal of Business* 38(1): 34–105.

Frank, MZ, and T Stengos. 1989. "Measuring the Strangeness of Gold and Silver Rates of Return." *Review of Economic Studies* 56(4): 553–67.

Gilmore, CG. 1996. "Detecting Linear and Nonlinear Dependence in Stock Returns: New Methods Derived from Chaos Theory." *Journal of Business Finance and Accounting*, 23: 1357–77.

Gilmore, CG. 2001. "An Examination of Nonlinear Dependence in Exchange Rates Using Recent Methods from Chaos Theory." *Global Finance Journal* 12(1): 139–51.

Greene, MT, and BD Fieltz. 1977. "Long-Term Dependence in Common Stock Returns." *Journal of Financial Economics* 4(3): 249–339.

Hamill, P, and KK Opong. 1997. "Nonlinear Determinism in the Irish Equity Market." Unpublished paper, School of Management, University of Ulster, Newtonabbey, 34–63.

Hampton, J. 1996. "Rescaled Range Analysis: Approaches for the Financial Practitioner, Part 3." *Journal of Neuro Vest Journal* 4(4): 27–30.

Hinich, MJ. 1982. "Testing for Gaussianity and Linearity of a Stationary Time Series." *Journal of Time Series Analysis* 3(3): 169–76.

Howe, JS, W Martin, and B Wood Jr. 1997, March. "Fractal Structure in the Pacific Rim." Southwestern Finance Annual Meeting, New Orleans.

Hsieh, DA. 1991. "Chaos and Nonlinear Dynamics: Application to Financial Markets." *The Journal of Finance* 46(5): 1839–77.

Hsieh, DA. 1993. "Implications of Nonlinear Dynamics for Financial Risk Management." *Journal of Financial and Quantitative Analysis* 28(1): 41–64.

Iseri, M, H Caglar, and N Caglar. 2008. "A Model Proposal for the Chaotic Structure of Istanbul Stock Exchange." *Chaos Solitons & Fractals* 36(5): 1392–98.

Lin, K. 1997. "The ABC's of BDS." *Journal of Computational Intelligence in Finance* 97(July/Aug): 23–26.

Liu, T, CWJ Granger, and WP Heller. 1993. "Using the Correlation Exponent to Decide Whether an Economic Series Is Chaotic." *Nonlinear Dynamics, Chaos and Econometrics*, John Wiley & Sons.

Madhavan, V. 2013a. "Nonlinearity in Investment Grade Credit Default Swap (CDS) Indices of US and Europe: Evidence from BDS and Close-Returns Tests." *Global Finance Journal* 24(3): 266–79.

Madhavan, V. 2013b. "Investigating the Nature of Nonlinearity in Indian Exchange Traded Funds (ETFs)." *Managerial Finance* 40(4): 395–415.

Mandelbrot, B. 1964. *The Variation of Certain Speculative Prices: The Random Character of Stock Prices.* MIT Press.

McKenzie, M. 2001. "Chaotic Behavior in National Stock Market Indices, New Evidence from the Close Returns Test." *Global Finance Journal* 12(1): 35–53.

McLeod, AI, and WK Li. 1983. "Diagnostic Checking ARMA Time Series Models Using Squared Residual Autocorrelations." *Journal of Time Series Analysis* 4(4): 269–73.

Mishra, R, S Sehgal, and N Bhanumurthy. 2011. "A Search for Long-Range Dependence and Chaotic Structure in Indian Stock Market." *Review of Financial Economics* 20(2): 96–104.

Opong, KK, G Mulholland, AF Fox, and K Farahmand. 1999. "The Behavior of Some UK Equity Indices: An Application of Hurst and BDS Tests." *Journal of Empirical Finance* 6(3): 267–82.

Papaioannou, G, and A Karytinos. 1995. "Nonlinear Time Series Analysis of the Stock Exchange: The Case of an Emerging Market." *International Journal of Bifurcation and Chaos* 5: 1557–84.

Peters, EE. 1989. "Fractal Structure in the Capital Markets." *Financial Analyst Journal* 45: 33–37.

Peters, EE. 1991a. *Chaos and Order in the Capital Markets: A New View of Cycles, Prices, and Market Volatility*, Vol. 1. John Wiley & Sons.

Peters, EE. 1991b. “A Chaotic Attractor for the S&P 500.” *Financial Analyst Journal* 47(2): 55–62.

Peters, EE. 1992. “R/S Analysis Using Logarithmic Returns.” *Financial Analyst Journal* 48(6): 81–82.

Peters, EE. 1994. *Fractal Market Analysis: Applying Chaos Theory to Investment and Economics,* Vol. 24. John Wiley & Sons.

Scarlat, E, C Stan, and C Cristescu. 2007. “Chaotic Features in Romanian Transition Economy as Reflected onto the Currency Exchange Rate.” *Chaos, Solitons & Fractals* 33(2): 396–404.

Scheinkman, J, and B LeBaron. 1989. “Nonlinear Dynamics and Stock Returns.” *Journal of Business* 62: 311–37.

Serletis, A, D Gogas, and P Calgary. 1997. “Chaos in East European Black-Market Exchange Rates.” *Research in Economics* 51(4): 359–85.

Serletis, A, and M Shintani. 2003. “No Evidence of Chaos But Some Evidence of Dependence in the US Stock Market.” *Chaos Solitons & Fractals* 17(2–3): 449–54.

Willey, T. 1992. “Testing for Nonlinear Dependence in Daily Stock Indices.” *Journal of Economics and Business* 44(1): 63–76.